Math 155 2016S Day 1

(PS1)

What are mathematical models?

- Concern things that change
- Mathematical descriptions or predictions of how things change.

- Change is usually change with time

- But sometimes it is something else
  "how revenue or profit change with price of good (not time),"

- Usually what things change with time?

global CO_2 emissions	(economy/resources)
global average temperature	(climate science)
global oil consumption	(economy/resources)
earthquakes	seismology
plate tectonics	(geology)
weather	(meteorology)

- All of these things are modeled mathematically

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Process of Modeling
- Make assumptions (art)
- Use assumptions and math to make inferences (art and science)

Assumptions often simplify the process you are trying to model.

Sometimes assumptions leave the model very simple
(that is going to be the case for us most of the time (elementary math models))

Sometimes the models are still very complicated (think weather models for forecasting)

Sometimes when the goal is understanding how a complicated system works - simplifying assumptions can leave the model simple enough to understand. Here insight is the goal.

The book is titled

"Elementary Mathematical Models Order aplenty with a glimpse of Chaos."

One of the course objectives is giving you a glimpse of chaos.

The book does this in the last chapter using math in a way that I think obscures the intuition.

If you are worried about understanding chaos, don't be. I am going to spend the next 10 minutes explaining it to you without math formulas - using analogy.

Students in the past have found my explanation easy to understand.

We are going to check this objective off the list today and I don't expect to come back to it.

My analogy is going to be kneading dough.

What do you do when you make bread?

Stretch / Fold / Mash Down.

Lets say you put a tiny drop of bright food coloring in the dough then kneed.

What is going to happen to drop as kneed?

Let's say you have a particular favorite molecule of food coloring which you are trying to keep track of.
You don't know where in the drop it is
But The drop is very small so you have a pretty good idea where it starts,

Do you have any way of knowing where in the bread dough it ends up? After lots of kneeds

OTOH if you have only one molecule of food coloring and you know it's in a very tiny region of the dough initially do you know where it ends up

but you don't know where in region.

This is really all there is to chaos. If you've understood what I said you've understood chaos.

Where does the math come in?

I promised not to use formulas But instead I'll use a visualization

You take the number line ← Do you know this?
Cut it down to every number greater than (equal) 0 less than (equal) 1.

Stretch / Fold / Mash Down

↑ exactly like kneading bread.

↑ so its same size as before

You can write a formula for this ← not going to
IF you start with a molecule
of food coloring (a number)

You know the number is in some
particular ~~tiny~~ region but you don't know exactly
where in the region

Kneed bread (apply function)
function gives you ~~a~~ a new number
(new position of food coloring molecule)

After a certain number of kneads
result is completely unpredictable
for the same reason as before

That completely weirded out people.

Apply to weather. Edward Lorentz
discovered chaos when he was studying
a model of weather. A small change
in position of food coloring (state
of atmosphere) has a profound change
on weather. Consider this: the route
you take to get here tonight is going
to change the path of hurricanes years
from now.

implications profound

What we are going to do in this class is work with sequences

I am going to give you a handout that introduces sequences

Pass out handout

After we know what a sequence is I am going to introduce a sequence that exhibits Chaos,

A very simple sequence.

Sometimes a sequence has a rule that determines terms.

Sometimes the rule tells you what one term is given a previous term

Other times given position.

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I am going to introduce a rule for terms of a sequence which is chaotic

Someone with a calculator tell me the digits of π?

~~I want a number between 0 and 1 so I am going to make an adjustment~~

1 term 3,1415....

next .1,415

next .4,15

e 2,71
7,15

always must be a number between 0 and 10

What if you knew the first term to 10 digits but didn't know the rest

(You've localized the molecule of food coloring to a tiny region)
Lets say you want to predict the first digit of 4 rain 5-9 Shine.
can do it for first 10 terms but no more. (in fact can be anything)

In fact can be anything.

You could write a formula for the shift (as called a shift)

You could also visualize what it is doing to number like

Stretch, cut into 10 pieces

instead of fold

mash together

cutting or folding give same thing

We are going to do much simpler rules

In book

- arithmetic
- quadratic
- geometric
- mixed
- logistic ← chaotic

we might not do all of them but as many as can