Math 151-2015XB-W3-Tues (Pg 1)

$\boxed{\begin{array}{l}\text{2.1 Review} \\ \text{2.1 New: Notation} \\ \qquad\qquad\text{Applications} \\ \text{2.2 New: Everything}\end{array}}$

Review: Functions

- Correspondence between two sets domain (first) and range (second) such that each element of domain maps to one and only one element of range.

Students $\rightarrow$ chairs

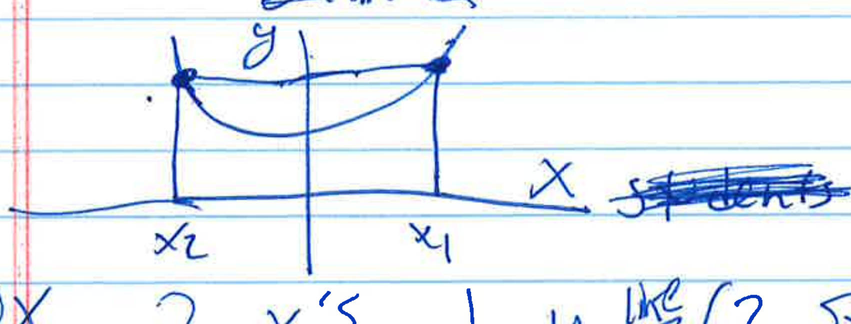
Two students in one chair: still a function

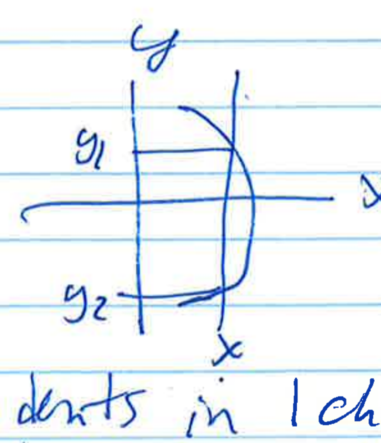
One student in two chairs; Not a function

When domain and range are real numbers you can graph function

~~Chair.~~

Vertical Line Test





OK 2 x's 1 y like (2 students in 1 chair)

1 x 2 y's (1 student in 2 chairs)

Notation: functions are denoted with a letter separate from variables

$f: y = 2x+1 \qquad g: y = x^2+2x+3$

Now we write

$f(3) = 2 \cdot 3 + 1 = 7$

Example 4: For $f(x) = \dfrac{12}{x-2}$

$g(x) = 1 - x^2$

$h(x) = \sqrt{x-1}$

Evaluate

(A) $f(6) = \dfrac{12}{6-2} = \dfrac{12}{4} = 3$

(D) $f(0) + g(1) - h(10)$

$= \dfrac{12}{0-2} + 1 - 1^2 - \sqrt{10-1}$

$= \dfrac{12}{-2} + 1 - 1 - \sqrt{9}$

$= -6 - 3 = -9$

matched

$f(-2)$

$$= \frac{12}{-2-2} = \frac{12}{-4} = -3$$

)

$$\frac{f(3)}{h(5)} = \frac{\frac{12}{3-2}}{\sqrt{5-1}} = \frac{\frac{12}{1}}{\sqrt{4}}$$

$$= \frac{12}{2} = 6$$

Finding Domains

(A) $F(x) = \dfrac{12}{x-2}$

OK unless divide by 0

not $x-2=0$ $x=2$

$x \neq 2$ Domain all real numbers except $x=2$

(B) $g(x) = 1 - x^2$

This is a number for all real numbers x

(C) $h(x) = \sqrt{x-1}$

Must have $x-1 \geq 0$

$x \geq 1$

Domain real numbers x where $x \geq 1$

or $[1, \infty)$

Matched 5

$F(x) = x^2 - 3x + 1$

$G(x) = \dfrac{5}{x+3}$

$H(x) = \sqrt{2-x}$

Find domains of F, G, H

F : all real numbers

G: $x = -3$

H: $2 - x \geq 0$

$-x \geq -2$

$x \leq 2$

$[-\infty, 2]$

~~Pg 32~~ Pg 6

Note from book: not always the same as
$f(x+h) \neq f(x) + f(h)$

This is ~~[illegible]~~ true for linear functions

but not Not ~~[illegible]~~ always for non linear function

f does not distribute,
Multiplication (starting notation) does
$2(x+h) = 2x + 2h$

Dont confuse

$f(\quad)$ with $2\cdot(\quad)$

Pg 7 Pg 8

Eg 6 for $f(x) = x^2 - 2x + 7$

find

(A) $f(a)$ $\quad a^2 - 2a + 7$

(B) $f(a+h) = (a+h)^2 - 2(a+h) + 7$

$= a^2 + 2ah + h^2 - 2a - 2h + 7$

(C) $f(a+h) - f(a)$

$\cancel{a^2} + 2ah + h^2 \cancel{-2a} - 2h \cancel{+7} - (\cancel{a^2} \cancel{+2a} \cancel{-7})$

$2ah + h^2 - 2h$

(D) $\dfrac{f(a+h) - f(a)}{h}$

$= 2a + h - 2$

9

Mathes Repeat for

$$f(x) = x^2 - 4x + 9$$

(A) $f(a)$

(B) $f(a+h)$

(C) $f(a+h) - f(h)$

(D) $\dfrac{f(a+h) - f(a)}{h}$

(A) $a^2 - 4a + 9$

(B) $(a+h)^2 - 4(a + h) + 9$

$\cancel{a^2} + 2ah + h^2 - \cancel{4a} - 4h \cancel{+9}$

(C) $-(\cancel{a^2} - \cancel{4a} + \cancel{9})$

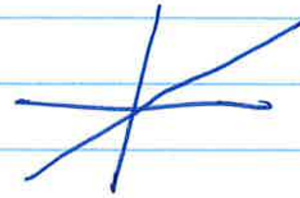
$2ah + h^2 - 4h$

(D) $2a + h - 4$

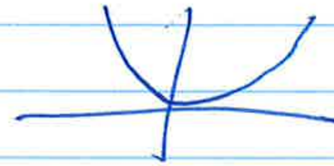
§2.2

A beginning library of elementary functions

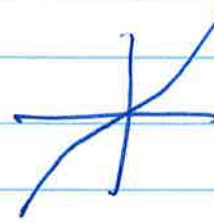
$f(x) = x$ identity function



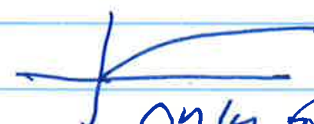
$h(x) = x^2$ square funct.



$m(x) = x^3$ cube function



$n(x) = \sqrt{x}$ square root function

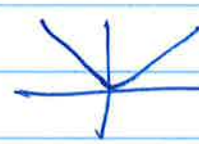


Only for $x \geq 0$

$p(x) = \sqrt[3]{x}$ cube root function

For all x

$g(x) = |x|$ absolute value function



Eg 1 Evaluate each at $x = 64$

$f(64) = 64$ $h(64) = 64^2 = 4096$

$m(64) = 64^3 = 262,144$

$n(64) = \sqrt{64} = 8$

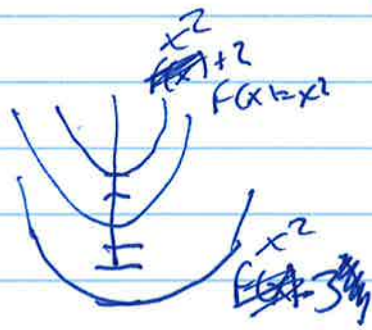
$p(64) = \sqrt[3]{64} = 4$

$g(64) = 64$

GRAPH TRANSFORMATIONS

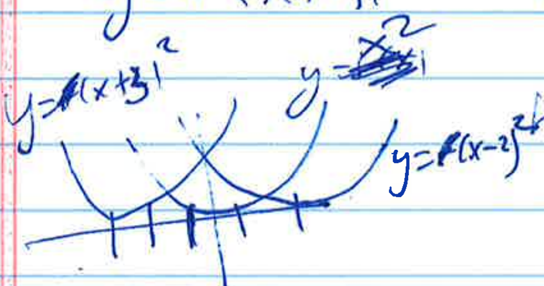
$y = f(x) + k$ — $k > 0$: shift graph of $y=f(x)$ up k units

$k < 0$: shift graph of $y=f(x)$ down k units

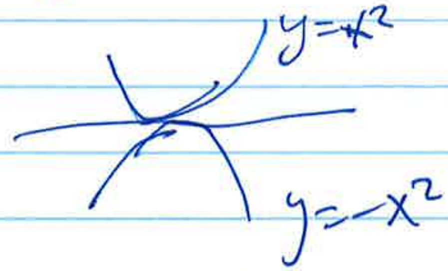


$y = f(x+h)$ — $h > 0$: shift graph of $y=f(x)$ left by h units

$h < 0$: shift graph of $y=f(x)$ right by h units

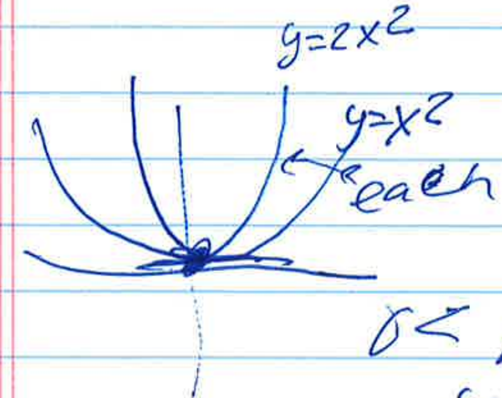


$y = -f(x)$ — Reflect graph of $y=f(x)$ about x-axis

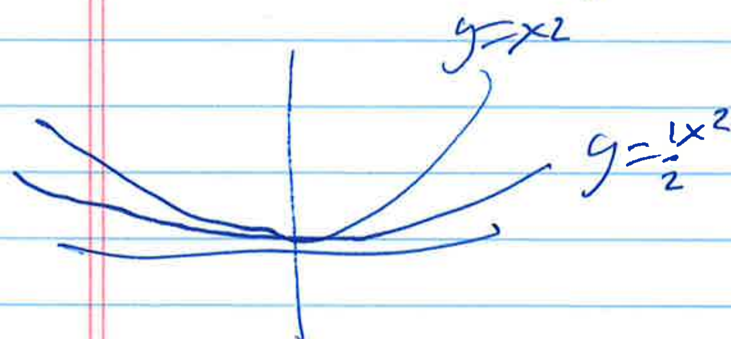


Vertical strech, shrink

$y = Af(x)$ $\quad A > 1$ strech graph of $y = f(x)$ vertically by multiplying each orginate by vale A



$0 < A < 1$ shrink ~~stretch~~ graph of $y = f(x)$ vertically by multiplying each ordinate by a vale A



Eg 2 m2
Eg 3 m3
Eg 4 m4
Eg 5

Explore and Discuss 1

(A) Vertical shift

(B) Horizontal shift

Let $f(x) = x^2$.

(A) Graph $y = f(x) + k$ for $k = -4, 0$, and 2 simultaneously in the same coordinate system. Describe the relationship between the graph of $y = f(x)$ and the graph of $y = f(x) + k$ for any real number k.

(B) Graph $y = f(x + h)$ for $h = -4, 0$, and 2 simultaneously in the same coordinate system. Describe the relationship between the graph of $y = f(x)$ and the graph of $y = f(x + h)$ for any real number h.

EXAMPLE 2 Vertical and Horizontal Shifts

(A) How are the graphs of $y = |x| + 4$ and $y = |x| - 5$ related to the graph of $y = |x|$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

(B) How are the graphs of $y = |x + 4|$ and $y = |x - 5|$ related to the graph of $y = |x|$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

SOLUTION

(A) The graph of $y = |x| + 4$ is the same as the graph of $y = |x|$ shifted upward 4 units, and the graph of $y = |x| - 5$ is the same as the graph of $y = |x|$ shifted downward 5 units. Figure 2 confirms these conclusions. [It appears that the graph of $y = f(x) + k$ is the graph of $y = f(x)$ shifted up if k is positive and down if k is negative.]

(B) The graph of $y = |x + 4|$ is the same as the graph of $y = |x|$ shifted to the left 4 units, and the graph of $y = |x - 5|$ is the same as the graph of $y = |x|$ shifted to the right 5 units. Figure 3 confirms these conclusions. [It appears that the graph of $y = f(x + h)$ is the graph $y = f(x)$ shifted right if h is negative and left if h is positive–the opposite of what you might expect.]

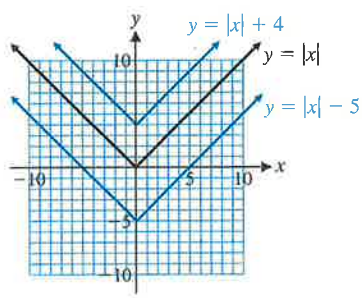


Figure 2 **Vertical shifts**

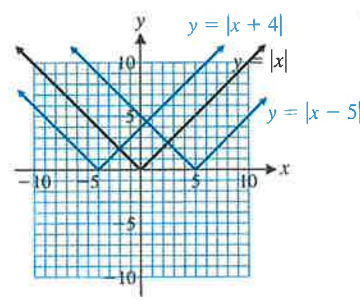


Figure 3 **Horizontal shifts**

Matched Problem 2

(A) How are the graphs of $y = \sqrt{x} + 5$ and $y = \sqrt{x} - 4$ related to the graph of $y = \sqrt{x}$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

(B) How are the graphs of $y = \sqrt{x + 5}$ and $y = \sqrt{x - 4}$ related to the graph of $y = \sqrt{x}$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

Comparing the graphs of $y = f(x) + k$ with the graph of $y = f(x)$, we see that the graph of $y = f(x) + k$ can be obtained from the graph of $y = f(x)$ by **vertically translating** (shifting) the graph of the latter upward k units if k is positive and downward $|k|$ units if k is negative. Comparing the graphs of $y = f(x + h)$ with the graph of $y = f(x)$, we see that the graph of $y = f(x + h)$ can be obtained from the graph of $y = f(x)$ by **horizontally translating** (shifting) the graph of the latter h units to the left if h is positive and $|h|$ units to the right if h is negative.

EXAMPLE 3 Vertical and Horizontal Translations (Shifts) The graphs in Figure 4 are either horizontal or vertical shifts of the graph of $f(x) = x^2$. Write appropriate equations for functions H, G, M, and N in terms of f.

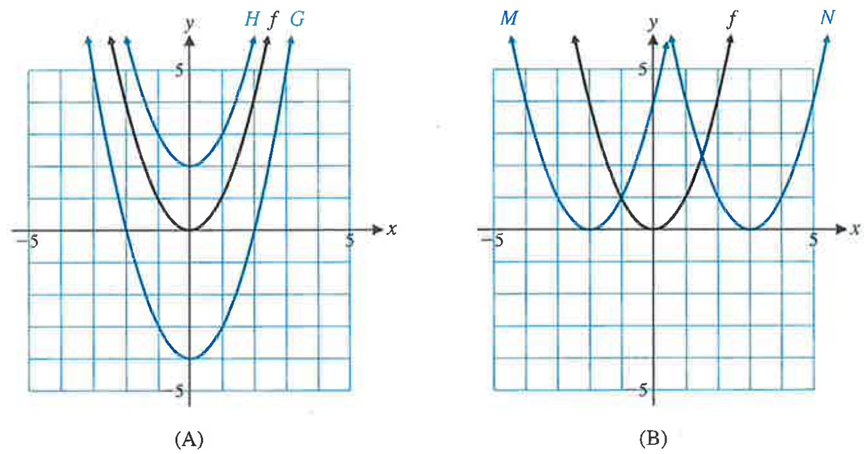


Figure 4 **Vertical and horizontal shifts**

SOLUTION Functions H and G are vertical shifts given by

$$H(x) = x^2 + 2 \qquad G(x) = x^2 - 4$$

Functions M and N are horizontal shifts given by

$$M(x) = (x + 2)^2 \qquad N(x) = (x - 3)^2$$

Matched Problem 3 The graphs in Figure 5 are either horizontal or vertical shifts of the graph of $f(x) = \sqrt[3]{x}$. Write appropriate equations for functions H, G, M, and N in terms of f.

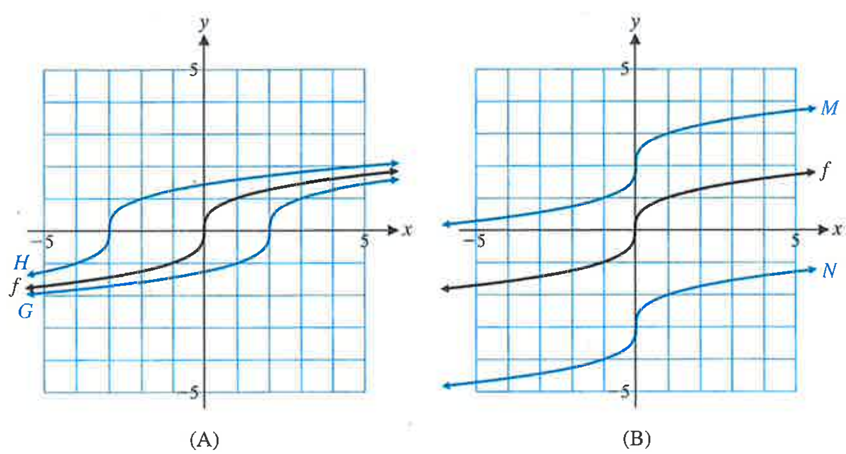


Figure 5 **Vertical and horizontal shifts**

Reflections, Stretches, and Shrinks

We now investigate how the graph of $y = Af(x)$ is related to the graph of $y = f(x)$ for different real numbers A.

Explore and Discuss 2

(A) Graph $y = Ax^2$ for $A = 1, 4$, and $\frac{1}{4}$ simultaneously in the same coordinate system.

(B) Graph $y = Ax^2$ for $A = -1, -4$, and $-\frac{1}{4}$ simultaneously in the same coordinate system.

(C) Describe the relationship between the graph of $h(x) = x^2$ and the graph of $G(x) = Ax^2$ for any real number A.

Comparing $y = Af(x)$ to $y = f(x)$, we see that the graph of $y = Af(x)$ ca be obtained from the graph of $y = f(x)$ by multiplying each ordinate value of th latter by A. The result is a **vertical stretch** of the graph of $y = f(x)$ if $A > 1$, **vertical shrink** of the graph of $y = f(x)$ if $0 < A < 1$, and a **reflection in th x axis** if $A = -1$. If A is a negative number other than -1, then the result is combination of a reflection in the x axis and either a vertical stretch or a verticа shrink.

EXAMPLE 4 Reflections, Stretches, and Shrinks

(A) How are the graphs of $y = 2|x|$ and $y = 0.5|x|$ related to the graph of $y = |x|$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

(B) How is the graph of $y = -2|x|$ related to the graph of $y = |x|$? Confirm your answer by graphing both functions simultaneously in the same coordinate system.

SOLUTION

(A) The graph of $y = 2|x|$ is a vertical stretch of the graph of $y = |x|$ by a factor of 2, and the graph of $y = 0.5|x|$ is a vertical shrink of the graph of $y = |x|$ by a factor of 0.5. Figure 6 confirms this conclusion.

(B) The graph of $y = -2|x|$ is a reflection in the x axis and a vertical stretch of the graph of $y = |x|$. Figure 7 confirms this conclusion.

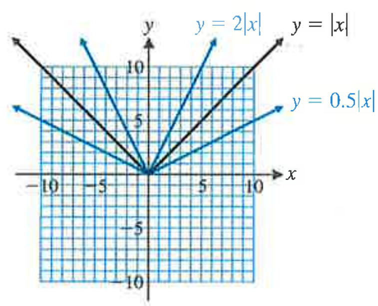


Figure 6 **Vertical stretch and shrink**

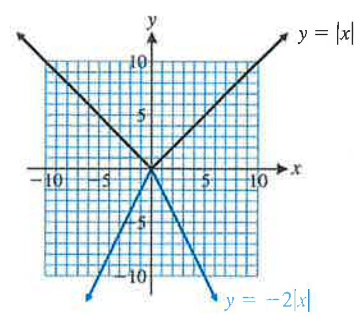


Figure 7 **Reflection and vertical stretch**

Matched Problem 4

(A) How are the graphs of $y = 2x$ and $y = 0.5x$ related to the graph of $y = x$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system. Vertical stretch; vertical shrink

(B) How is the graph of $y = -0.5x$ related to the graph of $y = x$? Confirm your answer by graphing both functions in the same coordinate system. Reflection in the x-axis and vertical shrink

The various transformations considered above are summarized in the following box for easy reference:

SUMMARY Graph Transformations

Vertical Translation:

$$y = f(x) + k \quad \begin{cases} k > 0 & \text{Shift graph of } y = f(x) \text{ up } k \text{ units.} \\ k < 0 & \text{Shift graph of } y = f(x) \text{ down } |k| \text{ units.} \end{cases}$$

Horizontal Translation:

$$y = f(x + h) \quad \begin{cases} h > 0 & \text{Shift graph of } y = f(x) \text{ left } h \text{ units.} \\ h < 0 & \text{Shift graph of } y = f(x) \text{ right } |h| \text{ units.} \end{cases}$$

Reflection:

$y = -f(x)$ Reflect the graph of $y = f(x)$ in the x axis.

Vertical Stretch and Shrink:

$$y = Af(x) \quad \begin{cases} A > 1 & \text{Stretch graph of } y = f(x) \text{ vertically by multiplying each ordinate value by } A. \\ 0 < A < 1 & \text{Shrink graph of } y = f(x) \text{ vertically by multiplying each ordinate value by } A. \end{cases}$$

Explore and Discuss 3 Explain why applying any of the graph transformations in the summary box to a linear function produces another linear function.

EXAMPLE 5 **Combining Graph Transformations** Discuss the relationship between the graphs of $y = -|x - 3| + 1$ and $y = |x|$. Confirm your answer by graphing both functions simultaneously in the same coordinate system.

SOLUTION The graph of $y = -|x - 3| + 1$ is a reflection of the graph of $y = |x|$ in the x axis, followed by a horizontal translation of 3 units to the right, and a vertical translation of 1 unit upward. Figure 8 confirms this description.

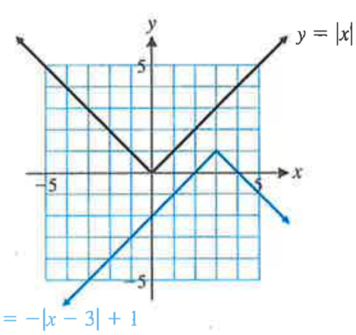


Figure 8 **Combined transformations**

Matched Problem 5 The graph of $y = G(x)$ in Figure 9 involves a reflection and a translation of the graph of $y = x^3$. Describe how the graph of function G is related to the graph of $y = x^3$ and find an equation of the function G.

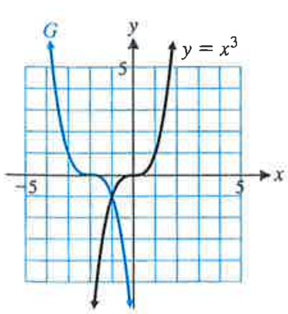


Figure 9 **Combined transformations**

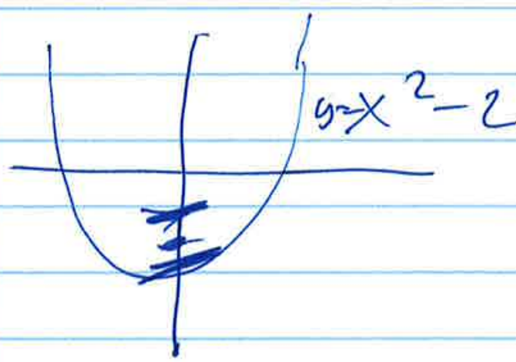
Piecewise defined functions

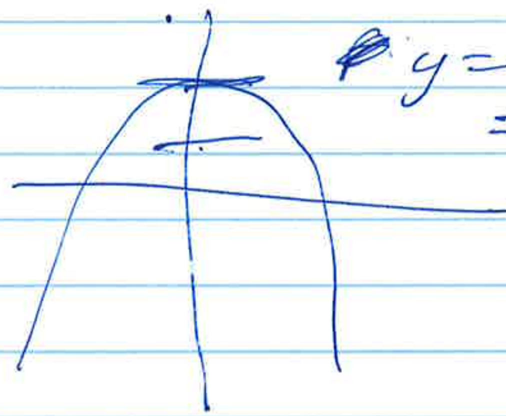
Recall absolute value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Functions can have different rules on different subsets of their domains

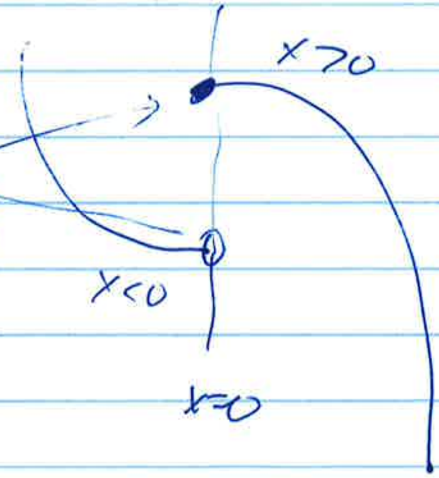
Function defined this way are called piecewise define. functions





$$y = -(x^2-2) = 2-x^2$$

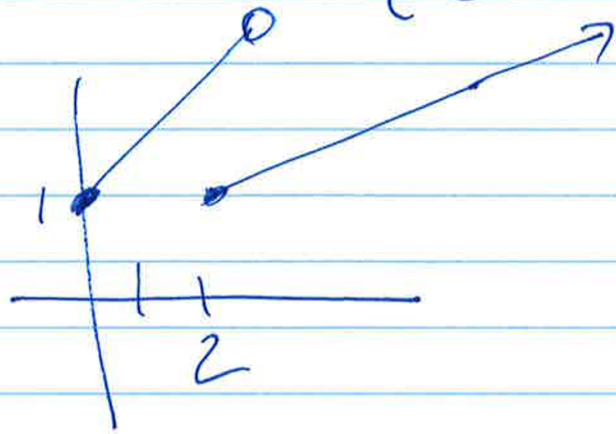
note endpoints



$$y = \begin{cases} x^2-2 & \text{if } x < 0 \\ 2-x^2 & \text{if } x \geq 0 \end{cases}$$

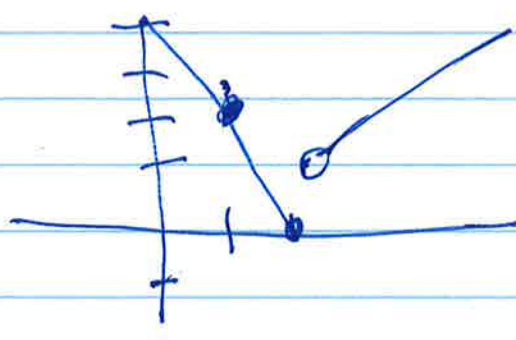
Pg 13

Graph $g(x) = \begin{cases} x+1 & 0 \le x < 2 \\ 0.5x & x \ge 2 \end{cases}$



Matched 6

Graph $h(x) = \begin{matrix} -2x+4 & 0 \le x \le 2 \\ x-1 & x > 2 \end{matrix}$



Natural gas is sold in units of CCF

price1 0.7866 / CCF for first 5 CCF
price2 0.4601 / CCF " next 35 CCF
price3 0.2504 / CCF " all over 40

x is number CCF sold

If $0 \le x \le 5$ x is number sold at price1

~~If $x > 5$ 5 sold at price 1~~

~~If x~~

~~If $x \le 5$ none sold at price 2~~

none sold a price 2
none sold at price 3

If $5 < x \le 40$
5 sold at price 1
$x-5$ sold at price 2
0 sold at price 3

If $x > 40$
5 sold at price 1
35 sold at price 2
$x-40$ sold at price 3

$$C(x) = \begin{cases} 0.7866x \\ (5)(0.7866) + (x-5)\,4.601 \\ (5)(0.7866) + (35)(4.601) + (x-40)(.2504) \end{cases}$$

Pg15

Matched 7

0.7675 for first 50
0.6400 for next 150
0.6130 for all over 200

For $x \le 50$
x sold @ 0.7675
0 " price 2
0 sold price 3

For $50 < x \le 200$
50 sold @ price 1
$x-50$ sold @ price 2
0 sold @ price 3

For $x \ge 200$
50 sold price 1
150 sold price 2
$x-200$ sold price 3

$$C(x) = \begin{cases} .7675x \\ (50)(.7675) + (x-50)(.6400) \\ (50)(.7675) + (150)(.6400) + (x-200)(.6130) \end{cases}$$