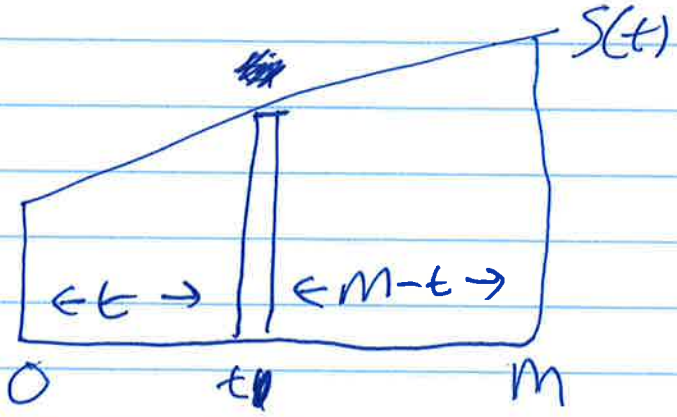
Math 211 – 2015S – W12 Friday

Review Present and future value: income streams

$B = Pe^{rt}$ future value

$P = Be^{-rt}$ present value



$$\text{Present value} = \int_0^M S(t)e^{-rt}\,dt = P$$

$$\text{Future value} = \int_0^M S(t)e^{(M-t)r}\,dt$$

$$= e^{Mr}\int_0^M S(t)e^{-rt}\,dt$$

$$= Pe^{rM}$$

New §6.4 Integrating Relative Growth Rate

Recall Relative rate of population growth

$$\frac{\text{rate of change}}{\text{population}} = \frac{P'(t)}{P(t)} = \frac{2000 \text{ people/year}}{100{,}000 \text{ people}} = 0.02 \text{ 1/years}$$

$$= 2\% \text{ per year}$$

$$\text{Relative Growth Rate} = \frac{1}{P}\frac{dP}{dt} = \frac{d}{dt}(\ln(P))$$

Using Fundamental theorem to integrate

$$\int_a^b \frac{P'(t)}{P(t)}\,dt = \int_a^b \frac{d}{dt}\ln(P(t))$$

$$= \ln(P(t))\Big|_a^b$$

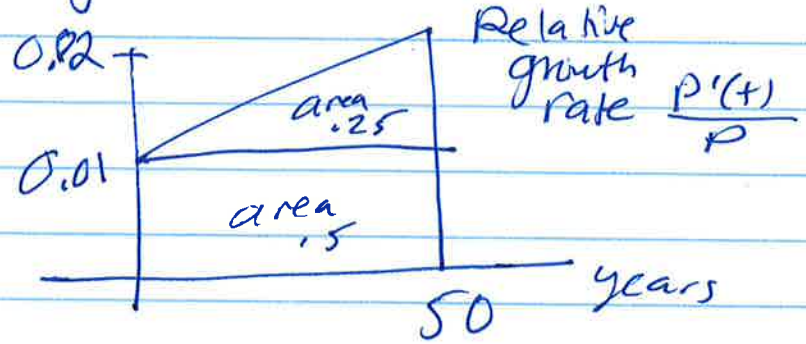
$$= \ln(P(b)) - \ln(P(a))$$

$$= \ln\left(\frac{P(b)}{P(a)}\right)$$

If you call integral I then $I = \ln\left(\frac{P(b)}{P(a)}\right)$

So $e^I = \frac{P(b)}{P(a)}$ or $\boxed{P(b) = e^I P(a)}$

Example: The relative rate of growth of a population is given by



Find factor that population increased

$$P(b) = P(a) \cdot e^{I}$$

$$I = \int \frac{P'(t)}{P(t)} dt = .75$$

Factor increase is $e^{.75} = 2.1$

Population increased 2.1 x times or just over doubled.