Math 211
Spring 2014
Exam 2–Practice
2/14/14
Time Limit: 75 Minutes

Name (Print): Answers

Study Guide #2

This exam contains 6 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	20	
2	30	
3	30	
4	20	
Total:	100	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1} \\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x \\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= 7\cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g+fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x)
\end{aligned}$$

1. (20 points) Compute the derivative of $y = x^x$. Hint: we had a formula when x was in the exponent with constant base ($y = a^x$) and another formula when x was in the base with constant exponent $y = x^n$. In this case x is in both the exponent and base, and neither formula will be useful. Instead, take the log of both sides, use the chain rule and product rule and solve for $\frac{dy}{dx}$.

$y = x^x \Rightarrow \ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

$\frac{dy}{dx} = y(\ln x + 1)$

$= x^x(1 + \ln x)$

$\boxed{\frac{dy}{dx} = x^x(1 + \ln x)}$

2. (30 points) The supply and demand curves are given by $q = 100 - 3p$ and $q = 2p - 50$. A sales tax of 8% is imposed.

(a) (10 points) Find the equilibrium price and quantity before the tax.

$100 - 3p = 2p - 50$

$150 = 5p$

$p^* = 30$

$q = 2 \cdot p - 50$

$= 2 \cdot 30 - 50$

$q^* = 60 - 50 = 10$

(b) (10 points) Find the new supply and demand curves after the tax.

sales tax on consumer applies to demand curve (the demand curve is the one with negative slope)

$100 - 3(1.08)p = 2p - 50$

$150 = [(3)(1.08) + 2]p$

Supply

$q = 2p - 50$

Demand

$q = 100 - 3(1.08)p$

(c) (10 points) Find the new equilibrium price and quantity.

$p = \frac{150}{2 + 3(1.08)} = \$28.63$ price before tax

$(1.08)(28.63) = \$30.92$ price after tax

$q = 2(28.63) - 50$

$q = 7.251$

3. (30 points) Take the following derivatives: (find $\frac{dy}{dx}$).

(a) (10 points)

$$y = \frac{e^{2x}}{1+x+e^x}$$

$$\frac{dy}{dx} = \frac{(1+x+e^x)e^{2x}\cdot 2 - e^{2x}(1+e^x)}{(1+x+e^x)^2}$$

(b) (10 points)

$$y = \sqrt{1+x+e^x} = (1+x+e^x)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(1+x+e^x)^{-1/2}(1+e^x)$$

(c) (10 points)

$$y = 4(\cos x)^3 + 3^x + 6 + 5e^{x^2} + 4\ln(\sin(x))$$

$$\frac{dy}{dx} = 12(\cos x)^2(-\sin x) + \ln(3)\,3^x + 0 + 5e^{x^2}\cdot 2x$$

$$\hookrightarrow + 4\frac{1}{\sin x}\cdot\cos x$$

4. (20 points) A bus company has fixed costs. For \$2 per ride the company attracts 500 passengers. With each additional \$0.10 the company loses 20 passengers (assume a linear demand eequation). How should the bus company set its price to maximize profits?

Write demand function

$$q = mp + q_0$$

$$q = \frac{\Delta q}{\Delta p} p + q_0$$

$$= \frac{-20}{.1} p + q_0$$

$$q = -200p + q_0$$

plug in $q = 500$ $p = \$2$

$$500 = (-200)(2) + q_0$$

$$500 = -400 + q_0$$

$$q_0 = 900$$

Demand equation

$$q = -200p + 900$$

Write Revenue

$$R = p \cdot q = p(-200q + 900)$$

$$= -200q^2 + 900q$$

Find Critical points $R' = -400q + 900 = 0$

$$q = \frac{900}{400} = \frac{9}{4} = 2.25$$

Check maximum $R'' = -400$ concave down, maximum

Bus company should raise price by \$0.25 to \$2.25 to maximize profits.

Second Additional Practice Problems

Exam 2

March 24, 2015

1. Compute:

$$\int (\sqrt{x} + 3\cos(3x) + e^{2x})dx$$

$= \frac{x^{3/2}}{3/2} + \sin(3x) + \frac{e^{2x}}{2} + C$

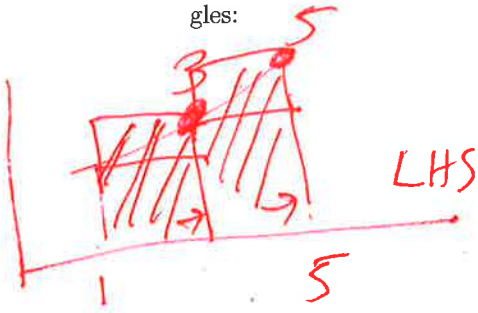
2. Compute:

$$\int_0^4 \sin(t)dt$$

$-\cos(t)\Big|_0^4 = \cos(4) - \cos(0)$

3. Estimate the following integral with a right-hand sum with $n = 2$ rectangles:

$$\int_1^5 (xe^x)dx$$



LHS $= 3e^3 \cdot 2 + 5e^5 \cdot 2$