Math 211-20155 - W4 - Friday

Review

Chain Rule - Applies to take derivative of a composite function

$$\left(F(g(t))\right)' = F'(g(t))\, g'(t)$$

If $y = F(z)$
and $z = g(t)$
then

$$\frac{dF}{dt} = \frac{dF}{dz}\frac{dz}{dt}$$

Examples

$$\frac{d}{dt}\left(5\ln(2t^2+3)\right) = \frac{5}{2t^2+3}\cdot 4t$$

$$\frac{d}{dx}\left(x^2+4\right)^3 = 3(x^2+4)^2(2x)$$

New

Product Rule

IF $u = F(x)$ and $v = g(x)$

Then $(Fg)' = F' \cdot g + F \cdot g'$ Newton

$\frac{d(uv)}{dx} = \frac{du}{dx} \cdot v + u\frac{dv}{dx}$ Leibniz

Examples

$$\frac{d}{dx}\left[x^2 e^{2x}\right] = 2x\, e^{2x} + x^2 2e^{2x}$$

$$\frac{d}{dt}\left[t^3 \ln(1+t)\right] = 3t^2 \ln(1+t) + t^3 \cdot \frac{1}{1+t}$$

$$\frac{d}{dx}\left[(3x^2+5x)e^x\right] = (6x+5)e^x + (3x^2+5x)\,e^x$$

We can now take the derivative of

$$y = x^x$$

Step 1

take logs

$$\ln y = \ln x^x$$

So $\ln y = x \ln x$

So $\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$$

$$\frac{dy}{dx} = y(\ln x + 1)$$

$$= x^x(\ln x + 1)$$

$$= x^x \ln x + x^x$$

Quotient Rule

IF $u = f(x)$ and $v = g(x)$

Then $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ (Newton)

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx} \cdot v - u \cdot \frac{dv}{dx}}{v^2}$$

Mnemonic I learned in high school

"low-d-high minus high-d-low

draw the line and square below"

Examples

$$\frac{d}{dx}\left[\frac{5x^2}{x^3+1}\right] = \frac{(x^3+1)(10x) - 5x^2(3x^2)}{(x^3+1)^2}$$

$$\frac{d}{dx}\left[\frac{1}{1+e^x}\right] = \frac{(1+e^x)\cdot 0 - 1\cdot(e^x)}{(1+e^x)^2}$$

$$= \frac{-e^x}{(1+e^x)^2}$$

$$\frac{d}{dx}\left[\frac{e^x}{x^2}\right] = \frac{x^2e^x - e^x(2x)}{x^4}$$

The book will simplify this to

$$\frac{x(e^x)(x-2)}{x^4}$$

$$= \frac{e^x(x-2)}{x^3}$$

§3.5 Periodic Functions Sin/Cos

$$\frac{d}{dt}\sin t = \cos t$$

$$\frac{d}{dt}\cos t = -\sin t$$

for chain Rule

$$\frac{d}{dt}\sin z = \cos z \frac{dz}{dt}$$

$$\frac{d}{dt}\cos z = -\sin z \frac{dz}{dt}$$

$$\frac{d}{dt}\sin kt = k\cos kt \qquad \frac{d}{dt}\cos kt = -k\cos kt$$

Examples

$$\frac{d}{dt}\left[\sin(t^2)\right] = (\cos(t^2)) \cdot (2t)$$

$$\frac{d}{dt}\, 5\cos(2t) = -10\sin(2t)$$

$$\frac{d}{dt}\, t\sin t = 1 \cdot \sin t + t\cos t$$

$$= \sin t + t\cos t$$