Math 211
Spring 2014
Exam 2
2/14/14
Time Limit: 75 Minutes

Name (Print): Answers

This exam contains 6 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	10	
2	20	
3	30	
4	40	
Total:	100	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$
\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1} \\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x \\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g+fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x)
\end{aligned}
$$

1. (10 points) Compute the derivative of $y = x^{2x}$. Hint: we had a formula when x was in the exponent with constant base ($y = a^x$) and another formula when x was in the base with constant exponent $y = x^n$. In this case x is in both the exponent and base, and neither formula will be useful. Instead, take the log of both sides, use the chain rule and product rule and solve for $\frac{dy}{dx}$.

$$y = x^{2x}$$

$$\ln y = \ln x^{2x}$$

$$= 2x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 2x \cdot \frac{1}{x} + 2 \cdot \ln(x)$$

$$\frac{dy}{dx} = y\left(\frac{2}{} + 2\ln x\right)$$

$$= x^{2x}\left(\frac{2}{} + 2\ln x\right)$$

2. (20 points) The demand and supply curves are given by $q = 100 - 4p$ and $q = 6p - 50$, respectively. A specific tax of \$1 is imposed on consumer.

(a) (10 points) Find the equilibrium price and quantity before the tax.

$$100 - 4p = 6p - 50$$
$$150 = 10p$$
$$p = 15$$
$$q = 100 - 4(15) = 40$$

(b) (10 points) Find the new supply and demand curves after the tax.

Old Demand Curve $q = 100 - 4p$

New Demand Curve $q = 100 - 4(p+1)$

Supply curve (doesn't change)

$$q = 6p - 50$$

3. (30 points) Take the following derivatives: (find $\frac{dy}{dx}$).

(a) (10 points)

$$y = \frac{2^{2x}}{1 + x^2 + \ln(x) + 3^x}$$

$$\frac{dy}{dx} = \frac{\left[\left(1+x^2+\ln(x)+3^x\right)(\ln 2)2^{2x}\cdot 2\right] - \left[2^{2x}\left(2x+\frac{1}{x}+\ln 3\,3^x\right)\right]}{\left[1+x^2+\ln(x)+3^x\right]^2}$$

(b) (10 points)

$$y = \frac{1}{\sqrt{1 + x^2 + e^x}} = (1+x^2+e^x)^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2}(1+x^2+e^x)^{-3/2}(2x+e^x)$$

(c) (10 points)

$$y = 4(\sin x)^3 + 6 + 5e^{3x} + 4\sin(\ln(x))$$

$$\frac{dy}{dx} = 12(\sin x)^2(\cos x) + 15e^{3x} + 4\cos(\ln(x))\frac{1}{x}$$

4. (40 points) The cost of producing a quantity q of an item is $C(q) = e^q$. The item sells for \$15.

(a) (10 points) What is the fixed cost?

$C(0) = e^0 = 1$

(b) (10 points) What is the marginal cost at $q = 1$?

$C'(q) = e^q$

$C'(1) = e^1 = e$

(c) (10 points) What value of q maximizes profit?

$\pi = R - C = 15q - e^q$

$\pi'(q) = 15 - e^q$

Critical points where $\pi'(q) = 0$

$e^q = 15 \Rightarrow \ln(e^q) = \ln 15$

$q = \ln 15$

(d) (10 points) How do you know your answer above is a maximum?

$\pi''(q) = -e^q$

$\pi(\ln(15)) = -e^{\ln(15)} = -15 < 0$

Concave Down

So. maximum

Additional Practice Problems

Exam 2

March 23, 2015

$$= \frac{x^4}{4} + \frac{3}{5}(-\cos(5x)) + e^x + C$$

1. Compute:

$$\int (x^3 + 3\sin(5x) + e^x)dx$$

$$\text{Integral} = \frac{x^4}{4} + \frac{3}{5}(-\cos(5x)) + e^x + C$$

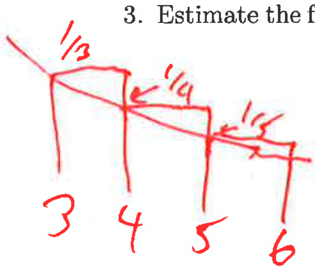
2. Compute:

$$\int_3^4 t^{-2}dt = \frac{t^{-1}}{-1}\Big|_3^4 = \frac{-1}{t}\Big|_3^4$$

$$\text{Integral} = \frac{-1}{4} + \frac{1}{3}$$

3. Estimate the following integral with a left-hand sum with $n = 3$ rectangles:

$$\int_3^6 (1/x)dx$$



$\Delta x = 1$

$$\text{LHS} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$