Math 211
Spring 2015
Exam 1
2/13/15
Time Limit: 75 Minutes

Name (Print): Answers

This exam contains 8 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. Graphing calculators should not be needed, but they can be used to check your work. If you use a graphing calculator to find an answer you must write the steps needed to find the answer, without the calculator.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	10	
4	10	
5	20	
6	20	
Total:	100	

1. (20 points) Consider the function $f(x) = x^2 \ln(x^2 + 1)$.

(a) (5 points) What is average rate of change of f between $x = 2$ and $x = 3$?

$$\frac{f(3) - f(2)}{3-2} = \frac{3^2 \ln(3^2+1) - 2^2 \ln(2^2+1)}{1}$$

$$= 9\ln(10) - 4\ln(5)$$

$$= 14.286$$

(b) (5 points) What is the instantaneous rate of change of f at $x = 2$?

$$f'(x) = x^2 \cdot \frac{1}{x^2+1} \cdot 2x + 2x \ln(x^2+1)$$

$$f'(2) = \frac{2^2 \cdot 2 \cdot 2}{5} + 4\ln(5) = 9.638$$

(c) (5 points) What is the relative rate of change of f at $x = 2$?

$$\frac{f'(2)}{f(2)} = \frac{\frac{16}{5} + 4\ln(5)}{4\ln(5)} = 3.8778$$

(d) (5 points) Is f concave up, concave down, or neither, at $x = 2$?

2. (20 points) The demand curve for a product is given by $q = 40 - 3p$. The supply curve is given by $q = 5 + 2p$.

(a) (5 points) Find the equilibrium price.

$$40 - 3p = 5 + 2p$$
$$35 = 5p$$
$$p^* = 7$$

(b) (5 points) Find the equilibrium quantity.

$$q^* = 40 - 3p = 40 - 3 \cdot 7 = 40 - 21$$
$$= 19$$

Check $q^* = 5 + 2p^* = 5 + 2 \cdot 7 = 5 + 14 = 19$

$$\boxed{q^* = 19}$$

(c) (5 points) Find the new equilibrium price if a \$1 per unit tax is charged to the consumer. List both the price the consumer pays per unit including the tax, and the amount the supplier receives per unit after the tax goes to the government.

$q = 40 - 3(p+1) = 40 - 3p - 3$

new demand curve $q = 37 - 3p$

supply doesn't change $q = 5 + 2p$

$37 - 3p = 5 + 2p$

$32 = 5p$

$p = \frac{32}{5} = 6.40$ price supplier receives

$p+1 = \frac{37}{5} = 7.40$ price consumer pays

(d) (5 points) Find the new equilibrium quantity if a \$1 per unit tax is charged to the consumer.

$q = 40 - 3(p+1)$

$= 40 - 3(7.40)$

$= 17.8$

3. (10 points) Find the equation of the tangent line to the function $f(x) = 5x^2 + x\ln(x)$ at $x = 1$.

Point on line $(x, f(x)) = (1, f(1))$ $f(1) = 5 + 1\cdot\ln(1) = 5$

$(1, 5)$ on line

Slope $f'(x) = 10x + \frac{x}{x} + \ln(x)$

$= 1 + 10x + \ln(x)$

$f'(1) = 1 + 10 + \ln(1) = 11$

Line $y = 5 + 11(x-1)$

$= 5 + 11x - 11$

$y = 11x - 6$

4. (10 points) The total cost of producing q units of a good is $C(q) = \ln(q+2)e^{2q}$. What are the fixed costs? What is the marginal cost of producing the 1^{st} unit of the good?

Fixed costs

$C(0) = \ln(0+2)e^0 = \ln(2) = .693$

Marginal cost of 1^{st} item

$C'(q) = \ln(q+2)2e^{2q} + \frac{1}{q+2}e^{2q}$

$C'(1) = \ln(3)\cdot 2e^2 + \frac{1}{3}e^2$

$= 18.698$

5. (20 points) The marginal cost of producing neato widgets is \$10 per widget (independent of quantity). The fixed cost is \$100. The widgets sell for \$20 each.

   (a) (5 points) Find the cost function relating total cost to number of widgets produced.

$$C(q) = 100 + 10q$$

   (b) (5 points) Find the revenue function relating revenue to number of widgets sold.

$$R(q) = 20q$$

   (c) (5 points) Find the profit function relating profit to number of widgets produced, assuming everything produced sells.

$$\pi(q) = R(q) - C(q) = 20q - (100 + 10q)$$
$$= 20q - 100 - 10q$$
$$= 10q - 100$$

   (d) (5 points) Find the break even point.

$$\pi(q) = 10q - 100 = 0$$
$$10q = 100$$
$$q = 10$$

6. (20 points) Take the derivatives of the following:

(a) (5 points) $y = \frac{x}{\ln(x)}$

$$\frac{dy}{dx} = \frac{\ln(x)\cdot 1 - x\cdot\frac{1}{x}}{(\ln(x))^2} = \frac{\ln(x)-1}{(\ln(x))^2}$$

(b) (5 points) $y = \sin(\cos(x))$

$$\frac{dy}{dx} = \cos(\cos(x))(-\sin(x))$$

(c) (5 points) $y = x^2e^{x^2}$

$$\frac{dy}{dx} = x^2e^{x^2}\cdot 2x + 2xe^{x^2}$$

(d) (5 points) $y = \frac{\sin(x)}{\cos(x)}$

$$\frac{dy}{dx} = \frac{\cos(x)\cos(x) + \sin(x)\sin(x)}{(\cos(x))^2}$$