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Math 151 - 2015XB - W2 Thurs

$\boxed{\S 1.2 - \text{Review}}$	+1, 3	+2, 1

Graphs and Lines
Cartesian Coordinate system
axes (x axis / y-axis) (horizontal / vertical)
quadrants
origin

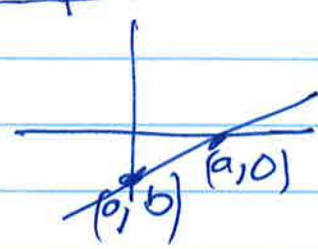
Questions?

Deep math - 1-1 correspondence between points in plane and ordered pairs
Also between lines in plane and solutions of equations in form $Ax + By = C$ A, B not both 0.
A solution is an ordered pair ~~that~~ (x, y) that satisfies equation

Questions?

<u>Solution set</u> is also called graph of equation

<u>Intercepts</u>



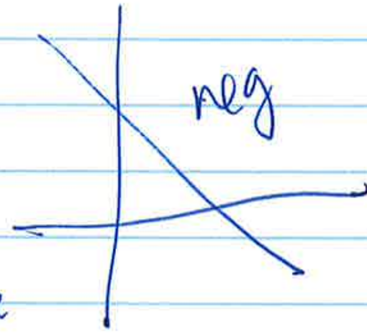
To ~~find~~ find x intercept plug $y=0$ in and solve for x

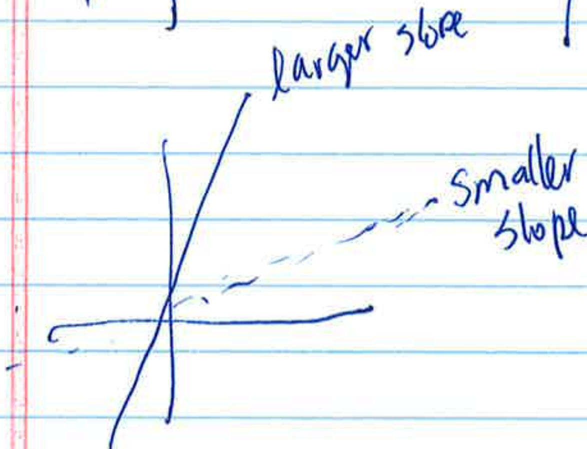
To find y-intercept plug $x=0$ in and solve for y

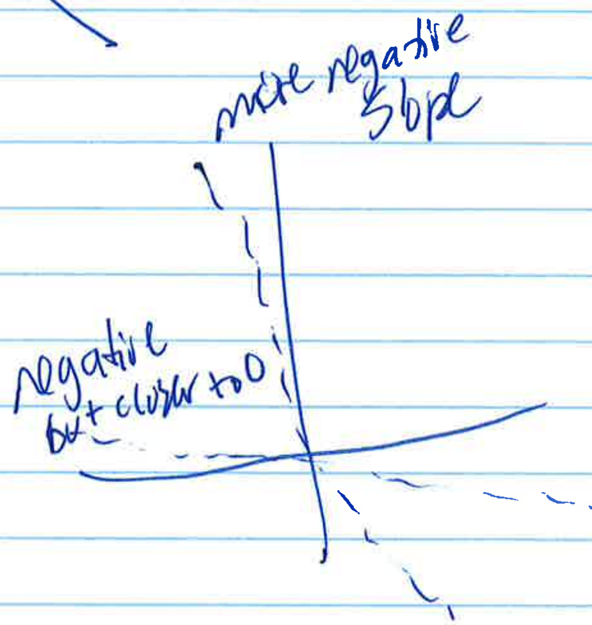
Graphing intercepts is a good way to graph line.

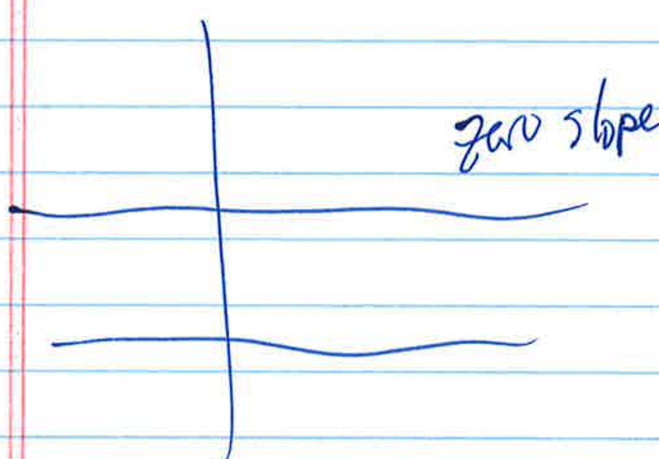
Vertical and Horizontal lines are different

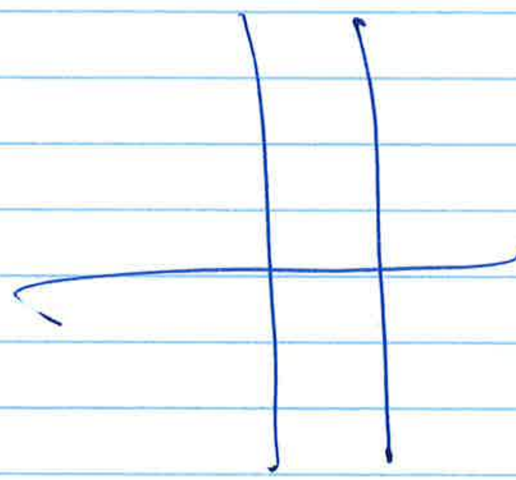
$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_1 - x_2}$$











Slope undefined

I. Slope $^{(m)}$ and Intercept $^{(b)}$

$$y = mx + b$$

II. Slope and a point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

III. Two points: (x_1, y_1) and (x_2, y_2)

Find slope with $\frac{y_2 - y_1}{x_2 - x_1} = m$

Use one of the points

IV. Two intercepts (not in book)

$$\frac{x}{a} + \frac{y}{b} = 1$$

After finding line you may be asked to put equation into one of the following forms

$$Ax + By = C$$

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

Applications from Tuesday

Costs $\underbrace{C}_{\text{total costs}} = \underbrace{b}_{\text{Fixed costs}} + mx$

mx is called the variable costs
m (slope) is called the marginal costs

Let's do an exercise.

Can you think of a good or service with high fixed costs and low variable costs.

How about high variable costs and low fixed costs,

Applications Supply and Demand

At one price supply and demand is ____
At another price supply and demand is ____

Q1: Find slopes of supply and demand equations

$$m_{supply} = \frac{price(2) - price(1)}{Supply(2) - Supply(1)} \qquad p = mx + b$$

$$m_{demand} = \frac{price(2) - price(1)}{demand(2) - demand(1)}$$

m_{Supply} should be positive
(the greater the price the more suppliers are willing to supply)

m_{demand} should be negative

Q2: Find supply and demand equations

$$p - p_1 = m(x - x_1) \quad \text{or} \quad p - p_1 = m(x - x_1)$$

price at that point
supply
supply at one point
demand
demand
price at that point.

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At a price of 12.59 per box of grapefruit, the supply is 595,000 boxes and demand is 650,000 boxes. At a price of $13.19 per box the supply is 695,000 boxes and the demand is 590,000 boxes. Assume linear relationships. Use units of 1000 boxes.

(A) Find price-supply equation

$$\frac{13.19-12.59}{695-595} = \frac{.60}{100} = 0.006$$

$$p-12.59 = 0.006(x-595)$$

$$p = 12.59 + 0.006x - (0.006)(595)$$

$$p = 0.006 + 9.02$$

(B) Find price-demand eqn

$$\frac{13.19-12.59}{590-650} = -0.01$$

$$p-12.59 = -0.01(x-650)$$

$$p-12.59 = -0.01x + 6.5$$

$$p = -0.01x + 12.59 + 6.5$$

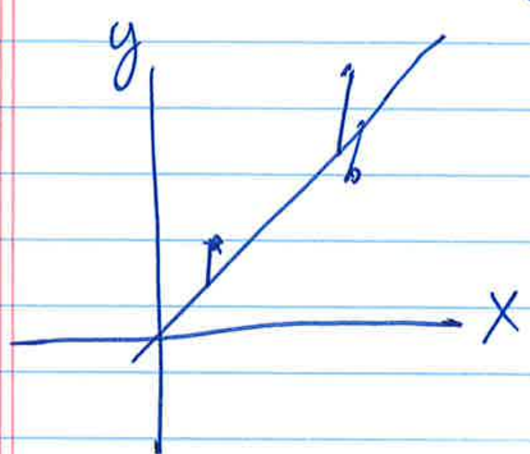
$$p = -0.01x + 19.09$$

Go over intercepts

Linear Regression §1.3

Go to textbook
Explain scatterplot and regression line

Linear least squares regression.



residuals

linear least square regression line minimizes the sum of squares of the residuals.

Efficient computation.

~~Existence Uniqueness (Slope m~~

Sensitive to outliers,

§2.1

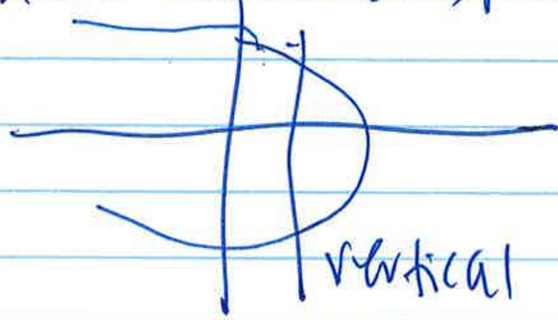
Functions – A function is a correspondence between two sets of elements such that to each element in the first set there corresponds one and only one element in the second set

The First Set is called the domain
The second set is called the range

range on vertical axis usually

domain usually on horizontal axis

An element in domain can't have two corresponding elements in range



vertical line test / not a function

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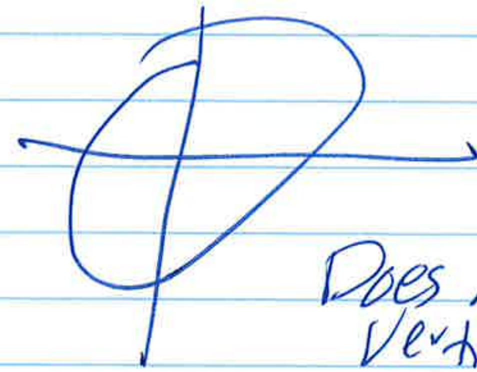
Equations in two variables

Sometimes define function but consider

$x^2 + y^2 = 1$



Graph is a circle

Does not obey vertical line test

To graph the solution set of an equation, Plug numbers in for x or y and solve for the other variable

This is tedious

Can be but not always numbers

domain

range

Functions define correspondences in real life
- to each person in this room there corresponds his/her anit
- to each employee there correspond an annual income
- to each item for sale there correspond a price
- to each student there corresponds a gpa.
- to each day there correspond a max temp
- For manufacture of x items there corresponds a cost $C(x)$
- For sale of x items, there corresponds a Revenue $R(x)$
- For each square there corresponds its area.
- To each number x there corresponds its cube x^3.

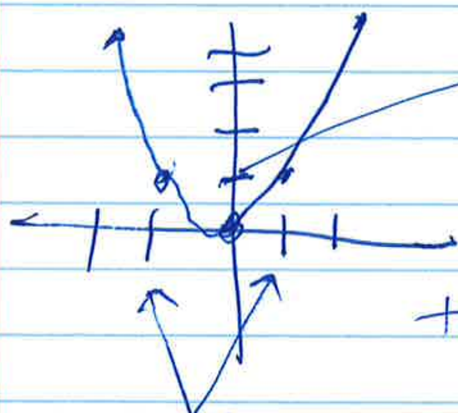
Number	Cube
-2	-8
-1	-1
0	0
2	8

correspondence (One-to-One)

Number	Square
-2	4
-1	1
0	0
1	1
2	4

Is this a function

Two from domain go to one in range.

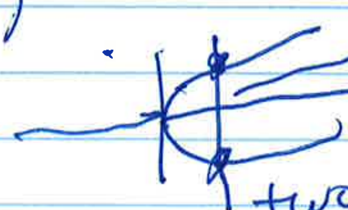


one in range

two to one function?

two in domain

You don't have a function when one in domain goes to two in range



one in domain

two in range

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Number → Square root

$0 \to 0$
$1 \to 1$
$1 \to -1$
$4 \to 2$
$4 \to -2$
$9 \to 3$
$9 \to -3$

$1^2 = 1$ / $(-1)^2 = 1$

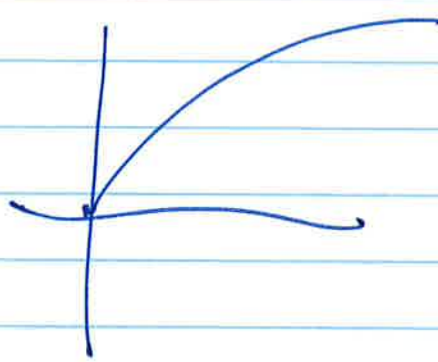
$\sqrt{1} = \pm 1$
$\sqrt{4} = \pm 2$
$\sqrt{9} = \pm 3$

Not a function

Unless you specify

positive square root (or 0)

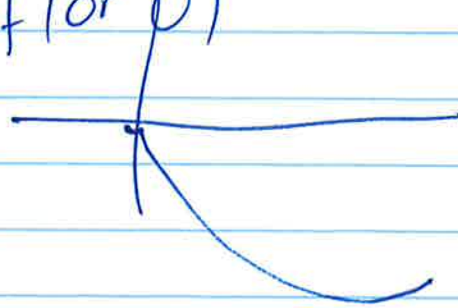
$0 \to 0$
$1 \to 1$
$4 \to 2$
$9 \to 3$



both are functions

or negative square root (or 0)

$0 \to 0$
$1 \to -1$
$4 \to -2$
$9 \to -3$

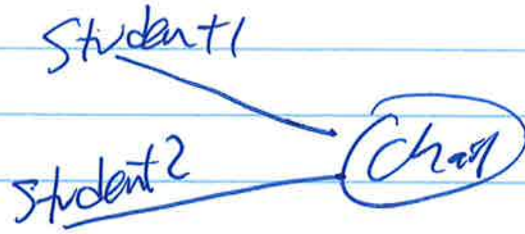


students in chairs again

Consider correspondence students to chairs they are sitting in.

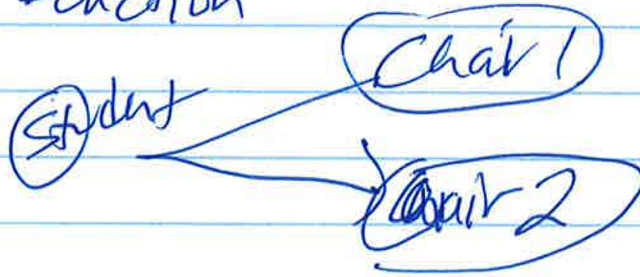
Domain = set of students
Range = set of chairs being sat in

If two students were sharing one chair would the correspondence be a function



Yes!

If one student was straddling two chairs would the correspondence be a function



No!

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Usually we will specify fuctions with equations domains and ranges will be real numbers (not students and chairs)

Correspondence between domain and range elements specified by an equation in x and y ~~where~~ set of values of x is domain and set of values for y is range.

$$y = x(x-1)$$

not 1-1

$x=1$ in domain corresponds to

$$y = 1(1-1) = 0 \text{ in range}$$

$x=0$ in domain corresponds to $y=0$ in range

$x=2$ in range corresponds to

$$y = 2(2-1) = 2(1) = 2$$

In range

function even though two values of x give same y.

Is this a function, No matter what you plug in for x get one value for y x is called the independent variable (domain)

y is called the dependent variable (range) because y depends on x

Example 2:

Determine which of following equations specify functions with independent variable x

(A) $4y - 3x = 8$
$\quad\;\; +3x \quad +3x$

$4y = 8 + 3x$

$y = 2 + \frac{3}{4}x$ yes! each x has one y solution

(B) $y^2 - x^2 = 9$

$y^2 = x^2 + 9$

~~[illegible]~~ $y = \pm\sqrt{x^2+9}$

NO! each x has two y solutions

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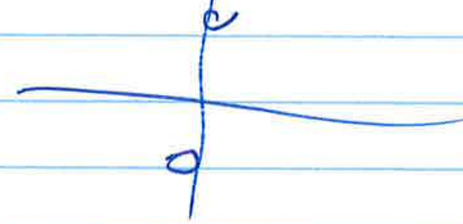
Matched 2

(A) $y^2 - x^4 = 9$



$y^2 = x^4 + 9$

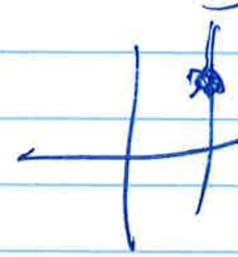
$y = \pm\sqrt{x^4+9}$ NO! two y's 1 x

(B) $3y - 2x = 3$

$3y = 3 + 2x$

$y = 1 + \frac{2}{3}x$ yes!!

1x goes to 1 y



Eg 3: Find domain of function specified by eqn $y = \sqrt{4-x}$

assuming x is independent variable and square root is positive square root

Usually $\sqrt{}$ is $+\sqrt{}$ pos square root

$-\sqrt{}$ is $-\sqrt{}$ neg square root

$\pm\sqrt{}$ is both

$4-x$ must be positive or 0
(we can't take square root of a negative number unless we allow for imaginary number (complex number) answers

Thus $4-x \geq 0$

$$-x \geq -4$$

$$x \leq 4$$

$$(\infty, 4]$$

matched 3

Find domain of function specified by equation

$$y = \sqrt{x-2}$$

$$x - 2 \geq 0$$

$$x \geq 2$$

$$[0, \infty)$$

Notation functions are denoted with a letter separate from variables

$f: y = 2x + 1 \qquad g: y = x^2 + 2x + 3$

Now we write

$$f(3) = 2 \cdot 3 + 1$$
$$= 7$$

Example 4: for $f(x) = \dfrac{12}{x-2}$

$g(x) = 1 - x^2$ and $h(x) = \sqrt{x-1}$

Evalute

(A) $f(6) = \dfrac{12}{6-2} = \dfrac{12}{4} = 3$

~~(D) f(0) + g(1) - h(10)~~

(D) $f(0) + g(1) - h(10)$

$= \left(\dfrac{12}{0-2}\right) + \left(1 - 1^2\right) - \sqrt{10-1}$

$= \dfrac{12}{-2} + 0 - \sqrt{9}$

$= -6 - 3$

$= -9$

Matches

A $F(-2)$

$$= \frac{12}{2+2} = 3$$

D $\frac{F(3)}{h(5)}$

$$\frac{\frac{12}{3-2}}{\sqrt{5-1}}$$

$$= \frac{12}{\sqrt{4}} = \frac{12}{2} = 6$$

Finding Domains

(A) $f(x) = \frac{12}{x-2}$ OK unless divide by 0

Unless $x-2=0$ or $x=2$

Domain all real numbers except $x=2$

(B) $g(x) = 1-x^2$

This is a number for all x

Domain all real number

(C) $h(x) = \sqrt{x-1}$

Real number if $x-1 \geq 0$

or $x \geq 1$

Domain $x \geq 1$ / $[1, \infty)$

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matched 5

$F(x) = x^2 - 3x + 1$

$G(x) = \frac{5}{x+3}$

$H(x) = \sqrt{2-x}$

Find domains

F: All reals

G: $x \neq -3$

H: $2 - x \geq 0$

$-x \geq -2$

$x \leq 2$ $(-\infty, 2]$

Note from book: not always the same as

$f(x+h) \neq f(x) + f(x)$

This is ~~[illegible]~~ true for linear functions

but not Not ~~[illegible]~~ always for non linear function

f does not distribute,

Multiplication (sharing notation) does

$2(x+h) = 2x + 2h$

Dont confuse

$f(\quad)$ with $2\cdot(\quad)$

Eg 6 for $f(x) = x^2 - 2x + 7$

Find

(A) $f(a)$ $\quad a^2 - 2a + 7$

(B) $f(a+h) = (a+h)^2 - 2(a+h) + 7$

$$= a^2 + 2ah + h^2 - 2a - 2h + 7$$

(C) $f(a+h) - f(a)$

$$\cancel{a^2} + 2ah + h^2 \cancel{- 2a} - 2h \cancel{+ 7} - \cancel{a^2} \cancel{+ 2a} \cancel{- 7}$$

$$2ah + h^2 - 2h$$

(D) $\dfrac{f(a+h) - f(a)}{h}$

$$= 2a + h - 2$$

Maths Repeat for

$$f(x) = x^2 - 4x + 9$$

(A) $f(a)$

(B) $f(a+h)$

(C) $f(a+h) - f(h)$

(D) $\dfrac{f(a+h) - f(a)}{h}$

(A) $a^2 - 4a + 9$

(B) $(a+h)^2 - 4(a + h) + 9$

$\cancel{a^2} + 2ah + h^2 - \cancel{4a} - 4h \cancel{+9}$

(C) $-(\cancel{a^2} - \cancel{4a} + \cancel{9})$

$2ah + h^2 - 4h$

(D) $2a + h - 4$