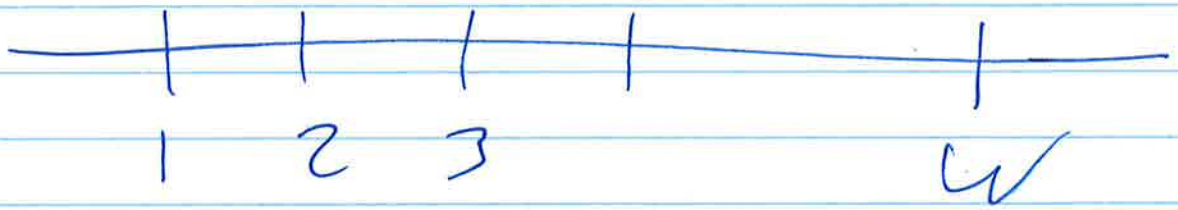
Math 151 2015XB Week 5 Thurs

Review

We had regular payments



And then withdrawal

Two problems yesterday

Put in n payments of PMT
At regular ~~intervals~~ (period)
Earn ~~[redacted]~~ interest rate i as a rate per period. The question is what is the value when we withdraw at the End of last period

$$FV = PMT \frac{(1+i)^n - 1}{i}$$

Fv stands for future value

The other type of problems is What PMT to put in to have FV in future

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

(sinking funds)

Both of these thing concern

FV the future value

Today we'll do PV present value

Today we are going to do something different. We are going to deposit a lump sum in our account today then withdraw the money at regular intervals. The question is what amount of money should we put in today to be able to withdraw at regular intervals.

Tuesday: we did future value of an annuity

Today: we do: present value of an annuity.

Key insight

For each payment

$$A = P(1+i)^n$$

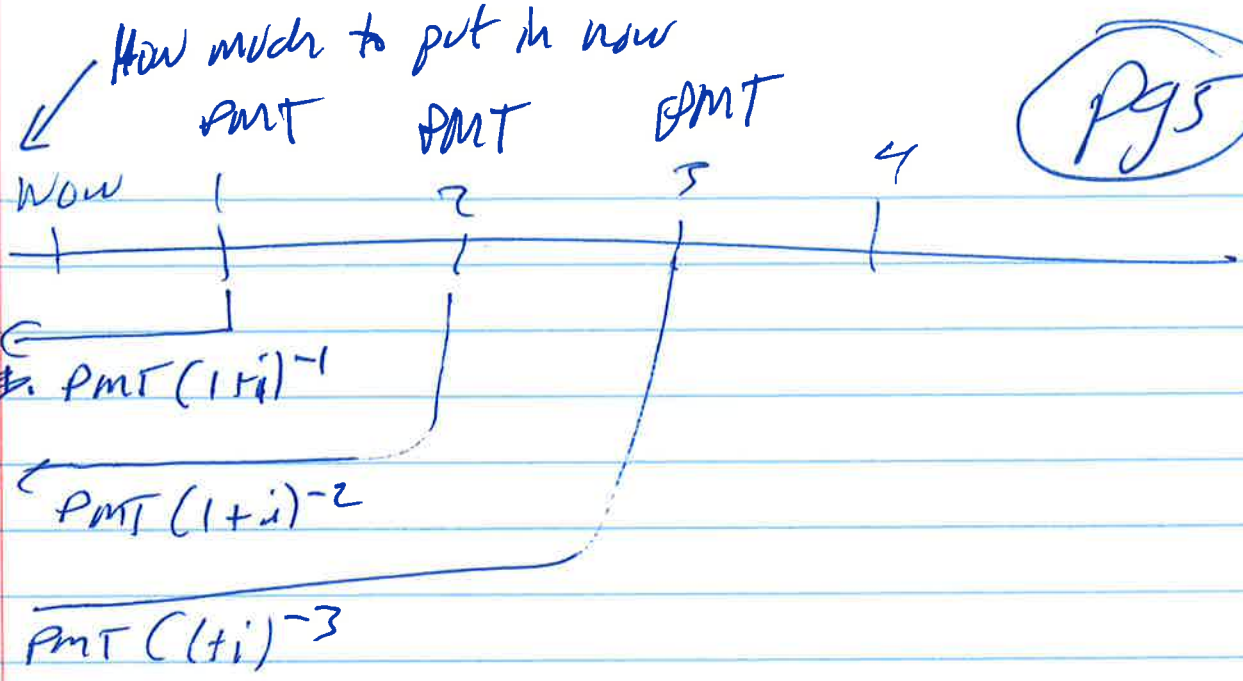
this is the formula for compound interest

- A: amount in account
- P: principal
- n: this varies depending on what payment you are talking about

A is the future value
P is the present value

Solve for P

$$P = \frac{A}{(1+i)^n} = A(1+i)^{-n}$$



Add these up and get the present value of the entire annuity (ie amount you need in account now to have enough for the payments).

So the PV of annuity is

$$PV = PMT(1+i)^{-1} + PMT(1+i)^{-2} + \cdots AMT(1+i)^{-n}$$

telescoping sum

$$(1+i)PV = PMT + PMT(1+i)^{-1} + \cdots + PMT(1+i)^{-n+1}$$

~~$PV - (1+i)PV = PMT$~~

$$(1+i)PV - PV = PMT - PMT(1+i)^{-n}$$

$$(1+i-1)PV = PMT(1-(1+i)^{-n})$$

$$PV = PMT\frac{(1-(1+i)^{-n})}{i}$$

Example 1

Amortization

For future value we had two formula

$$FV = PMT(\text{blah})$$

$$PMT = FV(\text{blah})$$

For present value we had one formula so far

$$PV = PMT(\text{blah})$$

Now we are going to have a second formula

$$PMT = PV(\text{blah})$$

p97

When is this useful?

When you have a certain amount in your account Now and you want to withdraw it over 5 years in equal payments every month, knowing interest rate per month i

Then use this formula

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

Also if you have a debt of $1000 = PV how much do you have to pay each month to amortize the debt

mortgage (kill it) Same formula

Example 3

We have 4 equations

$$FV = PMT \frac{(1+i)^n - 1}{i}$$

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

$$PV = pmt \frac{1-(1+i)^{-n}}{i}$$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

Sometimes you have to combine these formulas

Example 2