Math 151 - 2015XB - Week 5 - Mon

Last week: Exponential and Logaritmic Functions and some applications to finance and compound interest

Looking Forward: Chapter 3, "The Mathematics of Finance"

§3.1 Simple interest
§3.2 Compound and Continuous Compound interest - mostly a review.
§3.3 Future value of an Annuity Sinking Funds
§3.4 Present Value of An Annuity; Amortization

Let's start with Simple interest.

For simple interest you start with a principle P and you earn a fixed percentage of the principle every year - the interest doesn't earn interest unlike for compound interest

Money you start with is P

After one year it is $A = P + Pr$

principle

Fixed percentage of principle

After 1 year it is $P(1+r)$

That's the same with compound interest at the same rate compounded annually. But on the second year things change

Simple interest $A = P + Pr + Pr$

$= P(1+2r)$

After 3 years $A = P + Pr + Pr + Pr$

$= P(1+3r)$

After t years $A = P + Pr + Pr + \cdots + Pr$

$= P(1+tr)$

This is the formula you should know

$$\boxed{A = P(1+tr)}$$

For simple interest.

Compare with compound intrest

0 years: $A = P$ same

1 year: $A = P(1+r)$ same

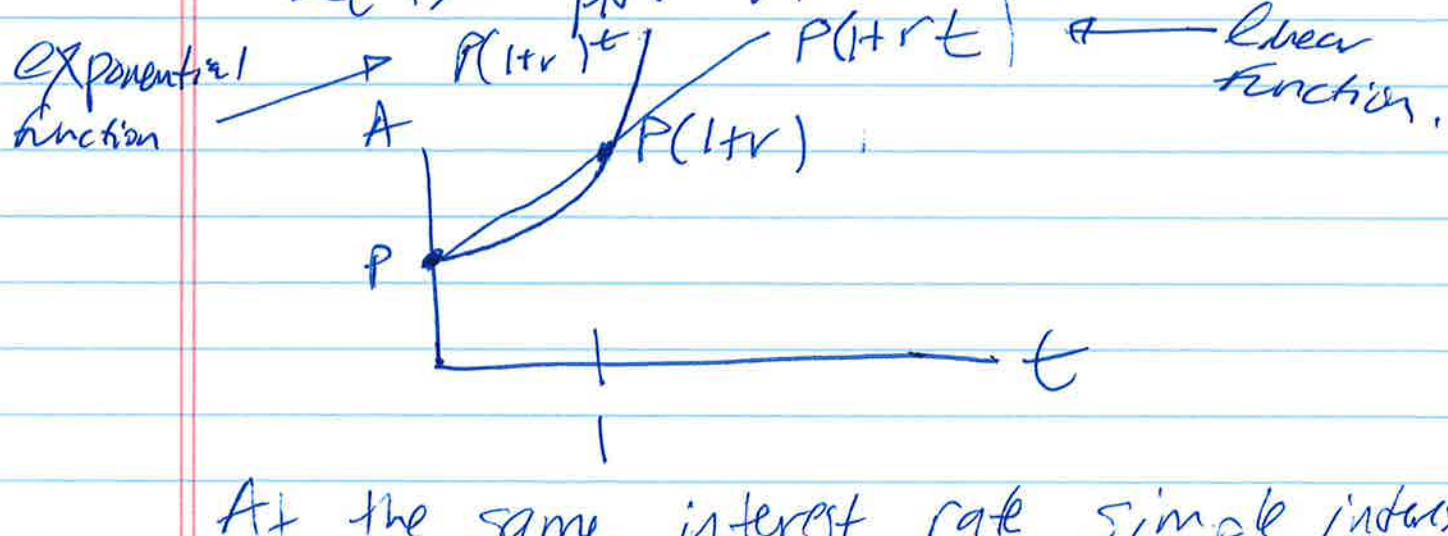
2 years: $A = P(1+r)(1+r)$
$= P(1+r)^2$

3 years: $A = P(1+r)(1+r)(1+r)$
$= P(1+r)^3$

t years: $A = P(1+r)\cdot(1+r)\cdot \ldots \cdot(1+r)$
$= P(1+r)^t$

Which is better simple interest or compound interest?

Lets plot it



At the same interest rate, simple interest is better for periods of less than a year while compound interest is better for periods of greater than a year.
In fact simple interest is only used for short periods typically < 1 yr.

Compound interest.

The formula I gave Thursday and Wednesday was

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

P was principal
r was the nominal annual interest rate.
m was number of times compounding per year
t was number of years

They give a different formula in chapter 3
Different but equivalent.

$$A = P(1+i)^{n}$$

$i = \frac{r}{m}$ is the rate per compounding period
i.e. monthly rate or daily rate.
not annual nomial rate
$n = mt$ is the number of compounding periods
Simpler formula but less interpetable parameter

Other formula is (same as Thurs) for continuous compounding

$A = Pe^{rt}$

P principal
r annual continuous interest rate
t is number of years

Annual percentage yield

On thursday I talked about the effective annual rate

for quarterly compounding, monthly compounding
daily compounding
Continuous compounding

What I said is that the effective annual rate is the rate that gives you the same amount of money when compounding annually as you get from whatever method the bank uses (quarterly, daily, continuous, etc).

Of course it is not the same as annual compounding, to the extent that you get the money in your account more frequently with more frequent compounding BUT after a year (or any integer number of years) it is the same, You get the same amount of money in your account on the anniversaries of the day you opened your account.

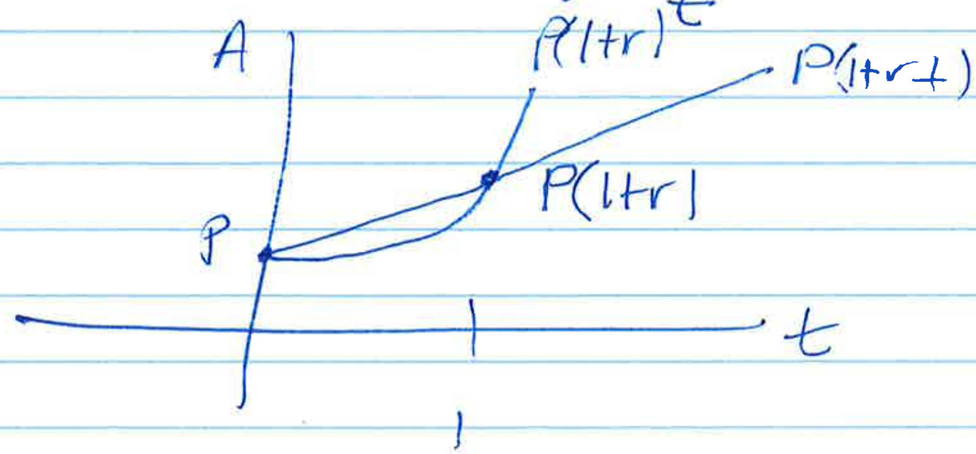
One source of caution if you read the textbook

The book defines the annual percentage yield (synonymous with effective annual rate) "as the

simple interest rate that will produce the same amount A in 1 year."

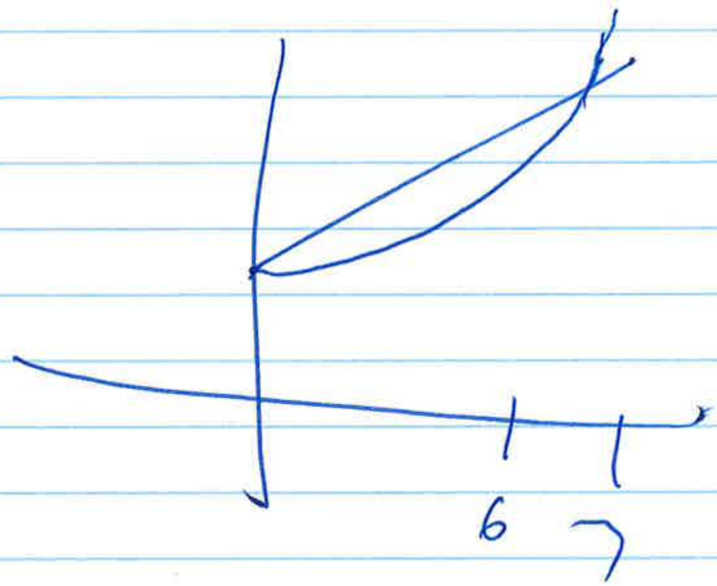
I defined it as the annual compounding rate that will produce the same amount A in 1 year.

Remember the graph



Both definitions (simple or annual compounding are the same).

Another word of caution for reading the book: the only place you see simple and compound interest plotted on the same axes,



The intersection point is not one.

Reading you see that one is for

9% simple interest and the other is for 7% compounded monthly.

If the interest rates are not the same they won't intersect at 1.

(Doesn't affect APY calculation though)

$7\frac{1}{2}$

nominal rates

Two formulas

$$APY = \left(1 + \frac{r}{m}\right)^m - 1$$

compounded m times per year

$$APY = e^{r} - 1$$ compounded continuously

Find by solving

$$(1 + r_{eff})^t = \left(1 + \frac{r_{nom}}{m}\right)^{mt}$$

$$(1 + r_{eff})^t = \left(e^{r_{nom}t}\right)$$

Take t^{th} root to get

$$1 + r_{eff} = \left(1 + \frac{r_{nom}}{m}\right)^m$$

$$1 + r_{eff} = e^{r_{nom}}$$

Subtract 1

Eg 1 §3.1

$$A = P(1 + rt)$$
$$= 500\left(1 + .12 \times \frac{30}{12}\right)$$
$$= 650$$

Eg 2 §3.1

$A = P(1 + rt)$ (P unknown)

$5000 = P[1 + (.1)(.5)]$ $r = .1$

$t = 1/2$ $(.75)$

$5000 = P[1.05]$

$$P = \frac{5000}{1.05} = 4751$$

Pg 9

Eg 3 §3.1

$A = P(1 + rt)$ (r unknown)

$10{,}000 = 9828.74\left(1 + r \cdot \frac{1}{2}\right)$

$10000 = 9828.74 + \frac{9828.74}{2} \cdot r$

$(10000 - 9828.74) \cdot \frac{2}{9828.74} =$

3.48%

Eg 4 §3.1

Find amount that will be paid

$A = P(1+rt)$

$= 3500\left(1 + (0.1)(270/360)\right)$

$= 3762.50$

$A = P + Prt$

$r = \frac{A-P}{Pt} = \frac{3762.5 - 3500}{(3500)\left(\frac{270-90}{360}\right)}$

$= .15$

15%

Eg 1 §3.2

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$t = 8$
$r = 0.06$
$m =$ varies
$P = 1000$

(a) $A = 1000(1+.06)^{8}$

(b) $A = 1000\left(1+\frac{0.06}{2}\right)^{8\cdot 2}$

(c) $A = 1000\left(1+\frac{0.06}{4}\right)^{8\cdot 4}$

(D) $A = 1000\left(1+\frac{0.06}{12}\right)^{8\cdot 12}$

$\boxed{\text{Eg2}}$

$t = 1.5$
$P = 8000$
$r = 0.09$
$m = 52$

$$A = 8000\left(1+\frac{.09}{52}\right)^{52\cdot 1.5}$$

$$A = 8000\, e^{0.09\cdot 1.5}$$

Eg3 §3.2

(A) $r=8\%$ $A=80000$ $P=$ unknown $m=2$ $t=17$

$$A=P\left(1+\frac{r}{m}\right)^{mt}$$

$$80000=P\left(1+\frac{0.08}{2}\right)^{2\cdot 17}$$

$$P=\frac{80000}{\left(1+\frac{0.08}{2}\right)^{2\cdot 17}}=\$21,084.17$$

(B) $A=Pe^{rt}$

$$P=Ae^{-rt}=80000\cdot e^{-(0.08)(17)}$$

$$\$20532.86$$

Pg 12

Eg 4 §3.2

$P = 10000$

$t = 10$

$A = 63{,}000$

$m = 1$

$A = P\left(1 + \frac{r}{m}\right)^{mt}$

$A = P(1+r)^{10}$

$63000 = 10000\,(1+r)^{10}$

$6.3 = (1+r)^{10}$

$\sqrt[10]{6.3} = 1+r$

$r = \sqrt[10]{6.3} - 1$

$= 20\%$

P917

$P = 10000$
$A = 25000$
$r = 0{,}08$
$m = 4$
$t =$ Unknown

$$A = P\left(1 + \frac{0{,}08}{m}\right)^{m \cdot t}$$

$$25000 = 10000(1 + 0.02)^{m \cdot t}$$

$$2{,}5 = 1{,}02^{4t}$$

$$\log(2{,}5) = 4t \log(1{,}02)$$

$$t = \frac{\log(2{,}5)}{4 \log(1{,}02)}$$

$$= 11{,}57 \text{ years}$$

Eg 6 § 3.2

APY

$$\left(1+\frac{0.0493}{12}\right)^2-1 = 5.043$$

$$APY=\left(1+\frac{r}{m}\right)^m-1$$

$$.058=\left(1+\frac{r}{4}\right)^4-1$$

$$1.058=\left(1+r/4\right)^4$$

$$\sqrt[4]{1.058}=1+r/4$$

$$4\left(\sqrt[4]{1.058}-1\right)=r$$

$$=0.056$$

5.6%