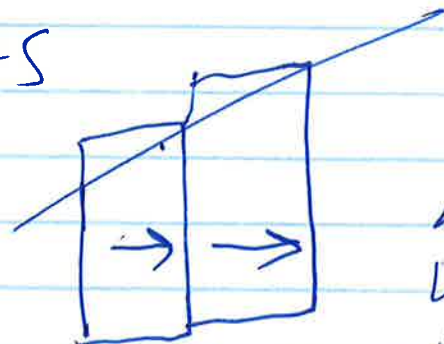
W8 Friday Problem Day

math 211-2015S-W9-Tuesday (Pg 1)

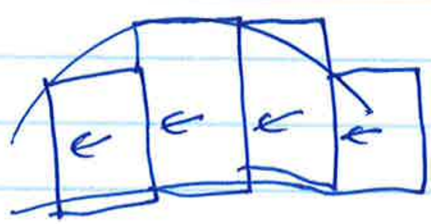
Review

We talked about formulas for the right-handed sum and the left-handed sum

RHS



Uses right endpoints of intervals to determine heights regardless of whether function is increasing / decreasing / or neither



LHS uses left endpoints

We also talked about an upper sum and a lower sum where the height of rectangles were respectively the max and min of the function on the interval spanned by the rectangle

Upper and lower sums were good for bounds, Right and left-handed sums were good for formulas.

Pg2

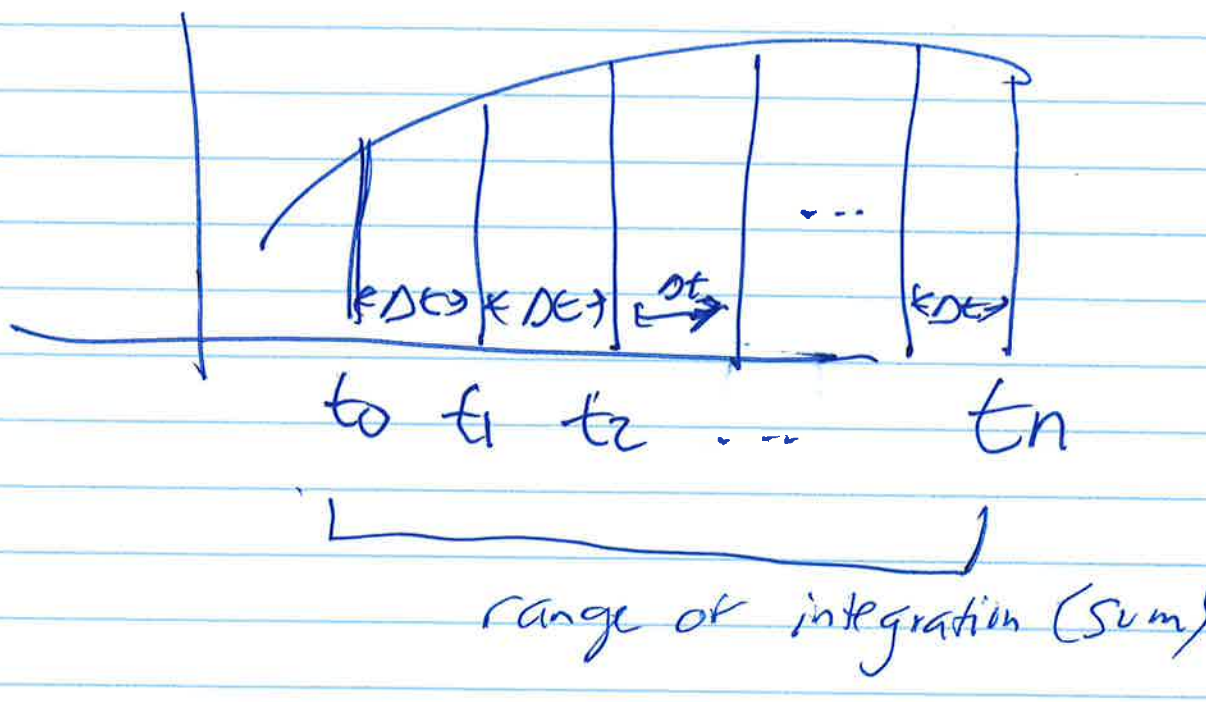
The formulas were

Right-handed sum

$$\sum_{i=1}^{n} f(t_i)\,\Delta t$$

Left-handed sum

$$\sum_{i=0}^{n-1} f(t_i)\,\Delta t$$



The limit of the sum as the number of rectangles n gets large and the width of the rectangles Δt gets small is the <u>integral</u>

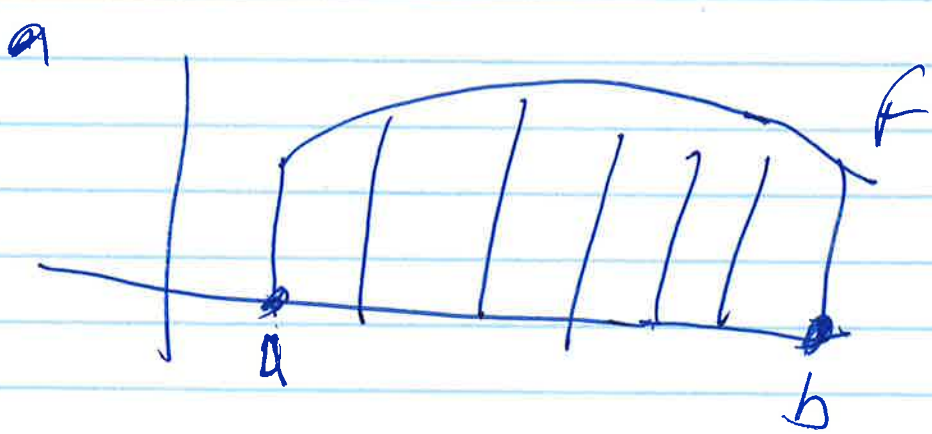
Sometimes called the Riemann Integral

The integral is written

$$\int_a^b f(t)\,dt$$

f is called the <u>integrand</u>

a and b are called the <u>limits of integration</u>

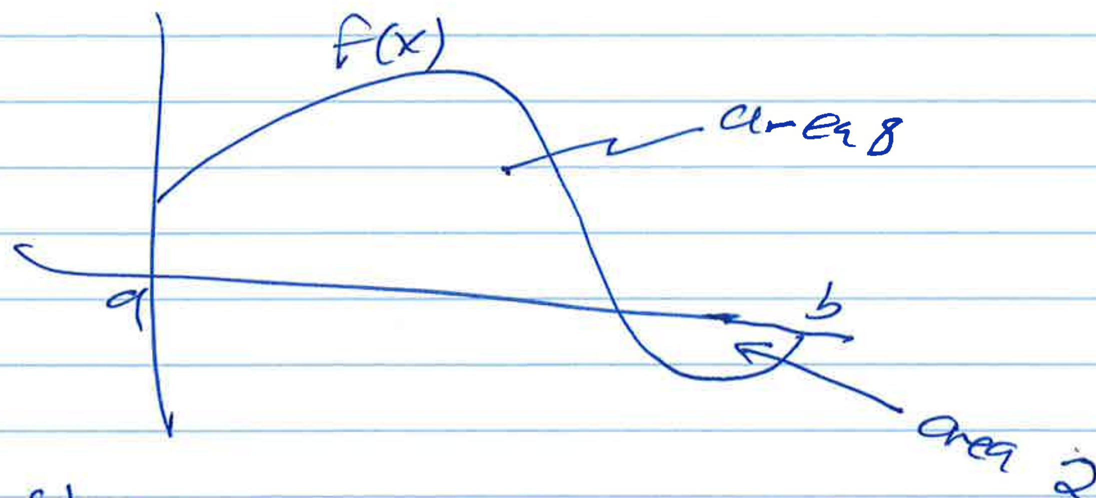


a is where the first rectangle starts and b is where the last rectangle ends.

The intgral is the area under the curve and above the x-axis when f is positive

Whre f is negative the area subtracts from the integral

Eg:



$$\int_a^b f(x)\,dx = 8 - 2 = 6$$

Units for the integral

These are the same as units for the Rieman sums

$$\sum_{i=1}^{n} \underbrace{f(x)}_{\text{Units of } f} \underbrace{\Delta x}_{\underbrace{(x_i - x_{i-1})}_{\text{Units of } x}}$$

The unit of measurement for

$\int_a^b f(x)\,dx$ is the product of

the units for $f(x)$ and the units for x

$$\int_a^b (\text{velocity})\,dt = \text{position}$$

$$\frac{\text{miles}}{\text{hr}} \cdot \text{hr} = \text{miles}$$

If x and $f(x)$ both have units of cm, then $\int_a^b f(x)\,dx$ has units of cm^2

Makes sense because this is an area.

Remember: If $f(t)$ is a rate of change of a quantity then

$$\int_a^b f(t)\,dt$$

represents the total change in the quantity between $t=a$ and $t=b$

Units correspond

$f(t)$ is a rate of change in a quantity, t is time, units are

$$\left(\frac{\text{quantity}}{\text{time}}\right) \times \text{time} = \text{quantity}$$

rate