Final Exam from last year
Some things on this exam we haven't covered! This is indicated.
Answers

Math 211
Spring 2014
Final Exam
4/2/14
Time Limit: 150 Minutes

Name (Print): ______________________

This exam contains 8 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. Graphing calculators should not be needed, but they can be used to check your work. If you use a graphing calculator to find an answer you must write the steps needed to find the answer, without the calculator.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	15	
8	30	
Total:	105	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1} \\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x \\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g+fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x) \\
\int x^n\,dx &= \frac{x^{n+1}}{n+1}+C, \quad n\neq -1 \\
\int \frac{1}{x}\,dx &= \ln(x)+C \\
\int e^{kx}\,dx &= \frac{1}{k}e^{kx}+C \\
\int (f(x)+g(x))\,dx &= \int f(x)\,dx+\int g(x)\,dx \\
\int cf(x)\,dx &= c\int f(x)\,dx \\
E &= \left|\frac{p}{q}\frac{dq}{dp}\right|
\end{aligned}$$

1. (10 points) The elasticity of a good is $E = 1.2$. What is the effect of a 1% increase of price on the quantity demanded.

This might be on exam.

$$E = \left|\frac{p}{q}\frac{dq}{dp}\right| = \left|\frac{\Delta q/q}{\Delta p/p}\right| = \frac{-x}{0.01} = 1.2$$

quantity demanded decreases by 1.2%

2. (10 points) The demand for a product is given by $q = 400 - 2p$. Find the elasticity of demand when the price is \$5.

This might be on exam

$$E = \left|\frac{p}{q}\frac{dq}{dp}\right|$$

$p = 5$

$q = 400 - 2 \cdot 5 = 390$

$\frac{dq}{dp} = -2$

$$E = \left|\frac{5}{390} \cdot -2\right| = \frac{10}{390} = \frac{1}{39} = 0.026$$

3. (10 points) Find the present and future value of an \$20,000 per year income stream over 15 years, with a discount rate of 4%.

4. (10 points) Find the average value of the function $f(x) = \frac{1}{x} + e^x$ within the interval $[0.2, 2.0]$.

5. (10 points) The marginal cost function of a product is $C'(q) = 3q^2 - 8q + 6$. The fixed costs are \$1000. Find the total cost to produce 10 items.

this might be on exam.

$C(b) = C(0) + \int_0^b C'(q)\,dq$

$C(10) = 1000 + \int_0^{10} (3q^2 - 8q + 6)\,dq$

$= 1000 + q^3 - 4q^2 + 6q \Big|_0^{10}$

$= 1000 + 1000 - 400 + 60 - 0$

$\boxed{1660}$

6. (10 points) The relative rate of growth of a population, between time t=0.5, and time t=3, is given by $r(t) = \frac{1}{x} + \frac{1}{x^2}$. Relate the population at time 0.5, to the population at time 3.

7. (10 points) Take the derivatives of the following:

these might be on exam

(a) (5 points) $y = e^{cos(x)}$

$$\frac{dy}{dx} = e^{\cos x}(-\sin x)$$

(b) (5 points) $y = \frac{sin(x)}{e^x}$

$$\frac{dy}{dx} = \frac{e^x \cos x - \sin x \, e^x}{(e^x)^2}$$

(c) (5 points) $y = x^7 cos(x)$

$$\frac{dy}{dx} = 7x^6 \cos x + x^7(-\sin x)$$

Problem 5 might be on exam

8. (30 points) The narrow gauge railroad between Durango, Colorado and Siverton, Colorado runs tourists between the two picturesque moutain towns. For \$10 per ride, the train attracts 100 passengers, and can take only one trip per day. For each additional dollar charged for the fare, the company loses 10 passengers. Assume a linear demand equation. The fixed costs for each journey are \$500. The marginal cost is zero (i.e. if the train is running, it doesn't cost anything more to take an additional passenger—but there must be an available seat on the train).

   (a) (10 points) How should the train company set the price of a ticket to maximize profits, assuming there are always enough seats?

$$q = mp + q_0 = \frac{\Delta q}{\Delta p} p + q_0 = \frac{-10}{1} p + q_0$$

$$q = -10p + q_0 \qquad 100 = -10 \cdot 10 + q_0 \Rightarrow q_0 = 200$$

$$q = -10p + 200$$

$$R(q) = p \cdot q = p(-10p + 200) = -10p^2 + 200p$$

$$R'(q) = -20p + 200 \qquad p = \frac{200}{20} = \$10$$

profits already maximized at \$10

Maximum because $R''(q) = -20 < 0$ concave down maximum

   (b) (10 points) How many seats should there be on the train allow profits to be maximized? If you haven't successfully solved part (a), full credit for (b) will be given for a formula for the answer to (b) in terms of the answer for (a).

$$q = -10p + 200$$
$$= -10 \cdot 10 + 200$$
$$= 100 \text{ seats}$$

(c) (5 points) What if the train can only take 10 passengers! Now how should the train company set its price to maximize profits? (Note: maximizing profits might still incur a loss—in which case, maximizing profit is the same as minimizing loss).

Demand Curve

$q = 10 = -10p + 200$

$-190 = -10p$

$p = \$19$

Set price to \$19

(d) (5 points) Bonus!!! If the number of passengers that can be accommodated is too small, the train company must operate at a loss. What is the minimum number of seats on the train needed to break even, assuming the price is set to maximize profits (same as: to minimize loss).

Fixed costs \$500 want Revenue ≥ 500

$R(p) = -10p^2 + 200p = 500$

so $p^2 - 20p + 50 = 0$

$p = \frac{20 \pm \sqrt{400 - 4 \cdot 50}}{2} = 10 \pm \sqrt{50} = \begin{cases} \$17.071 \\ \$2.93 \end{cases}$

← need few seats to make \$500

← need lots of seats to make \$500

Want minimum use \$17.071

$q = -10p + 200 = -10(\underbrace{10 + \sqrt{50}}_{17.071}) + 200$

$= 29.23$

Need 30 seats to break even.