Math 211 - 2015S - W2 - Fri

Review

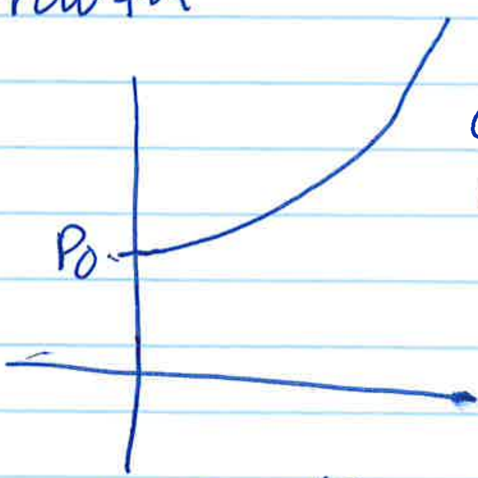
$f(x) = 7^x$ exponential function
$f(x) = x^7$ power law function

$f(x) = 7^x$
base is called the growth factor

$f(x) = a^x$
$r = a - 1$ percentage increase

$f(x) = 7^x$
what is percent increase? 600%

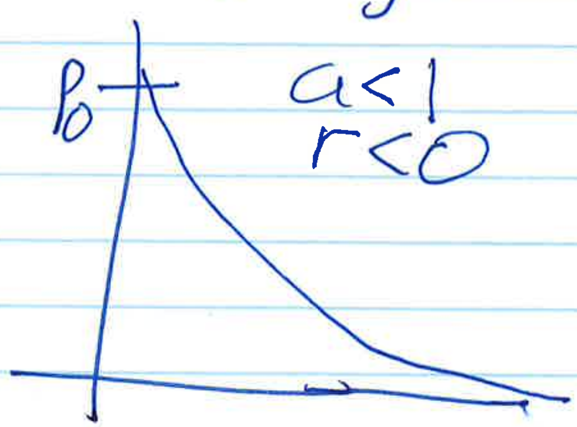
Exponential Growth



$a > 1$
$r > 0$

$P = P_0 a^t$

Exponential Decay



$a < 1$
$r < 0$

$P = P_0 a^t$

math 211-2015S-W2-Fri (Pg2)

There are two ways of writing an exponential function

$P=P_0 a^t$ a is called the growth factor

$P=P_0 e^{kt}$ k is called the continuous growth rate

$a=e^k$ equivalently,
$k=\ln a$

$$P=P_0 a^t = P_0 (e^k)^t = P_0 e^{kt}$$

In otherwords, you can write the exponential function in terms of the growth factor a or alternatively in terms of the continuous growth rate k
If you do this a and k are related by $a=e^k$ or $k=\ln a$.

New: Doubling time for exponential growth

$$P=P_0 e^{kt}$$
double initial $\rightarrow$ $2P_0=P_0 e^{kt}$
$$2=e^{kt}$$
$$\ln 2 = kt$$
$$t=\ln 2/k = \frac{\ln(2)}{\ln(a)}$$

Half life for exp decay

$$P=P_0 e^{kt}$$
$$\tfrac{1}{2}P_0 = P_0 e^{kt}$$
$$\tfrac{1}{2}=e^{kt}$$
$$\ln(1/2)=kt$$
$$t=\ln(1/2)/k = \frac{\ln(1/2)}{\ln(a)}$$

In both cases $t_{1/2}$ (t_2) independent of P_0.

Math 211-20155-W2-Fri

Financial Applications: Compound interest

P_0 is initial deposit in bank
$P(t)$ is balance at time t (no withdrawals just interest)

Compounded annually $P = P_0(1+r)^t$

r is annual interest rate (percentage increase)

Compounded continuously $P = P_0 e^{rt}$

r is continuous interest rate

It is a little confusing to use the same letter for both rates, but that is how book does it.

You can talk about the effective annual rate r_{eff} when the rate under continuous compounding is r

Solve ~~[illegible]~~ $1 + r_{eff} = e^r$ for r_{eff}

Composite Functions

$f(g(x))$

Eg $f(t)=t^2$ $g(t)=t+2$

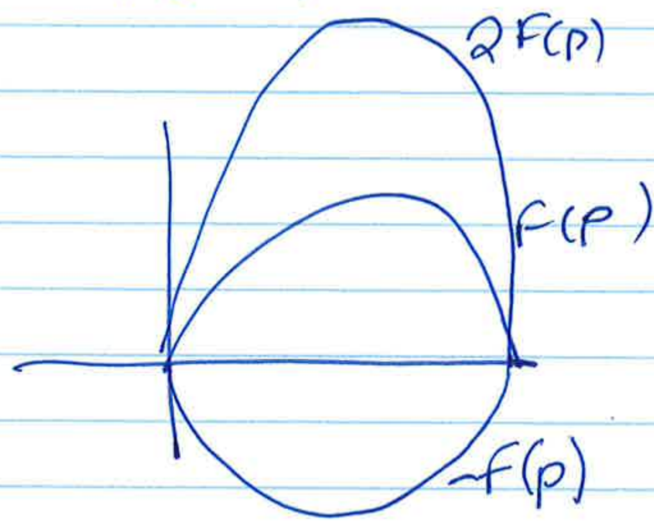
$f(t+1)=(t+1)^2$

$f(t)+3=t^2+3$

$f(g(t))=(t+2)^2$

$g(f(t))=t^2+2$

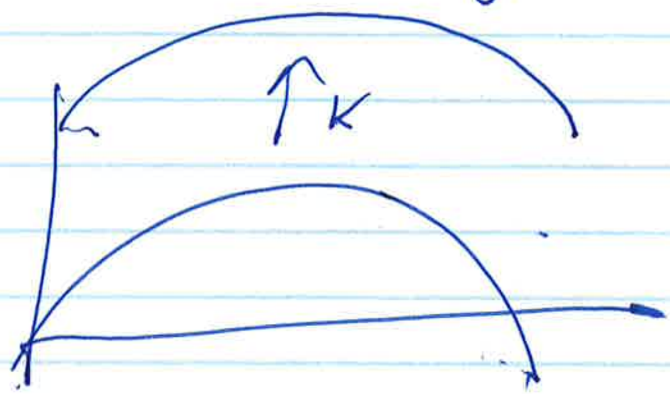
Multiplying by a constant stretches graph



Math 211 - 2015S - W2 - Fri (Pg 5)

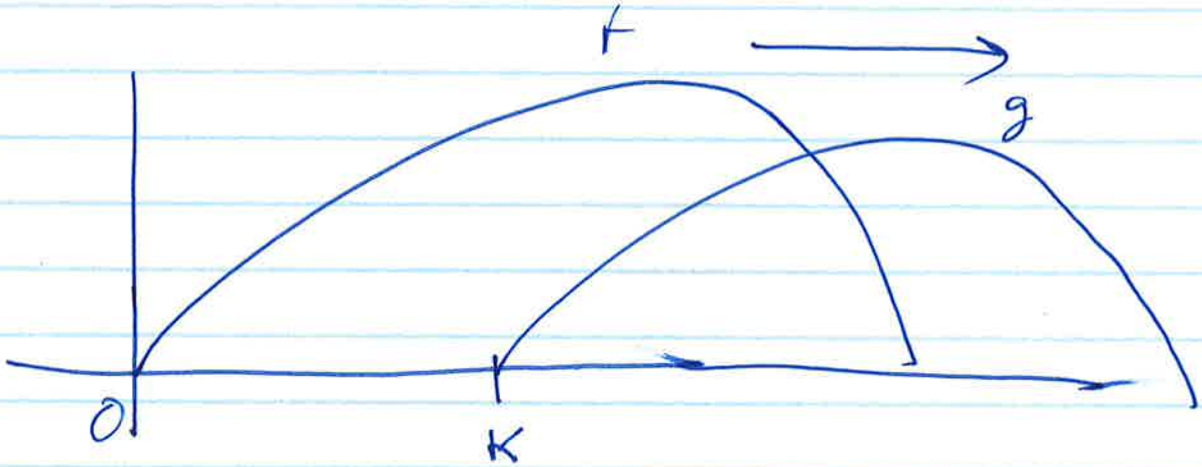
$y = f(x) + k$

moves f up by k



$y = f(x-k)$

moves f to the right by k units



$g(x) = f(x-k)$

$g(k) = f(k-k) = f(0)$

Math 211 - 2015 S - W2 - Friday Pg 6

Proportionality

We say y is directly proportional to x if there exists a non-zero constant k such that $y = kx$
(proportionality constant)

We say that $Q(x)$ is a power function if Q is proportional to a constant power of x

$$Q = k x^{p}$$

$\sqrt{x} = x^{1/2}$ power law

$x^{-1} = \frac{1}{x}$ power law.